Introduction to Robot Motion Planning & Navigation
Module 3

Solmaz S. Kia (solmaz.eng.uci.edu)

solmaz@uci.edu

Mechanical and Aerospace Engineering Department

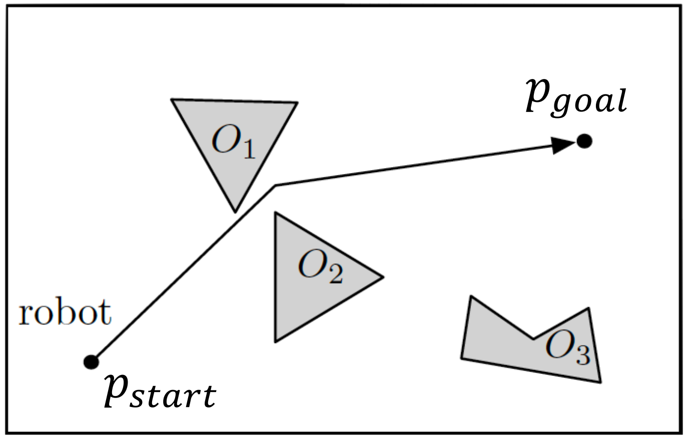
University of California Irvine

Chapter 3: Configuration Spaces

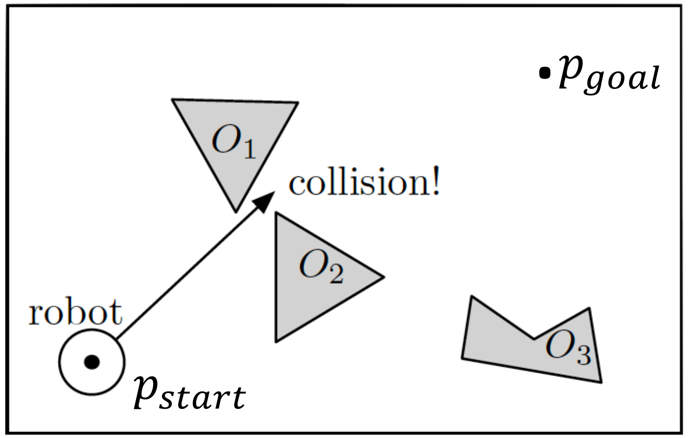
- describe a robot as a single or multiple interconnected rigid bodies,
- define the configuration space of a robot,
- examine numerous example configuration spaces, and
- discuss forward and inverse kinematic maps that arise in robot motion planning.

Motion Planning for Robots with Finite Shape and Size

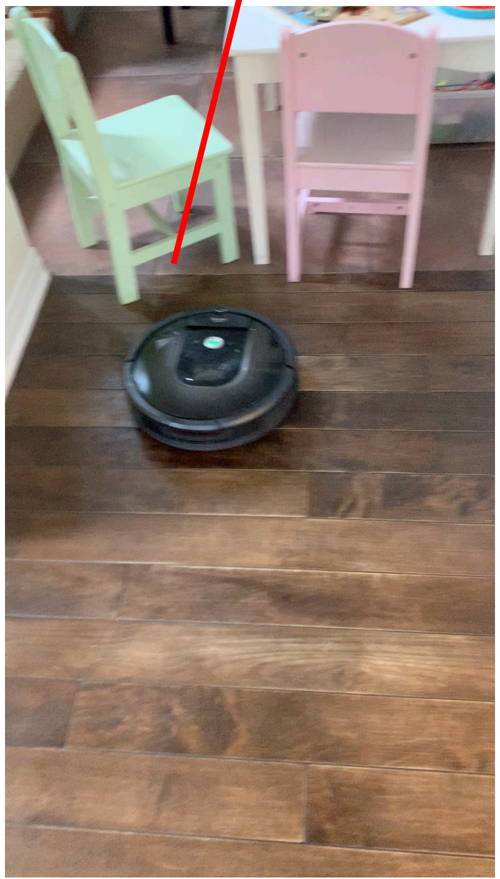
a robot described by a moving point (that is, the robot has zero size).



robots with a finite shape and size (a robot is composed of a single rigid body or multiple interconnected rigid bodies).



Robot cannot go through this space

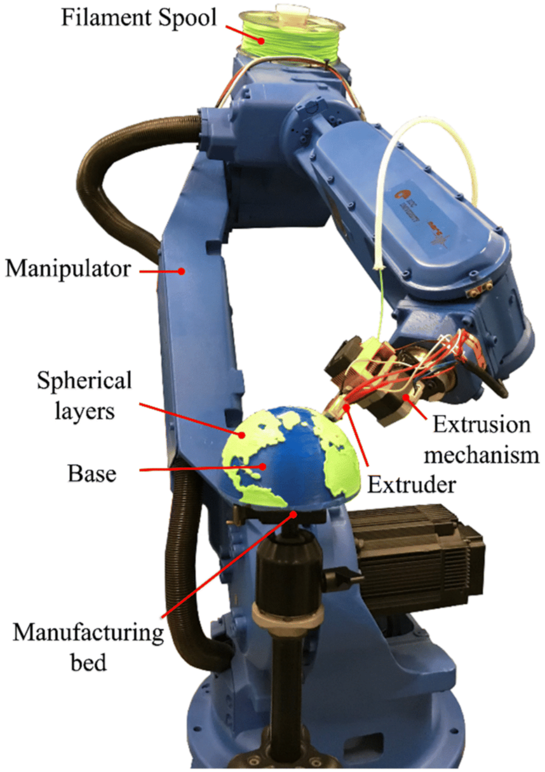


Robot Modeling

a rigid body is a collection of particles whose position relative to one another is fixed

A robot is composed of a single rigid body or multiple interconnected rigid bodies;

- robots are 3-dimensional in nature, but we will focus on planar problems



Soft robots also exist but not the focus of our study.

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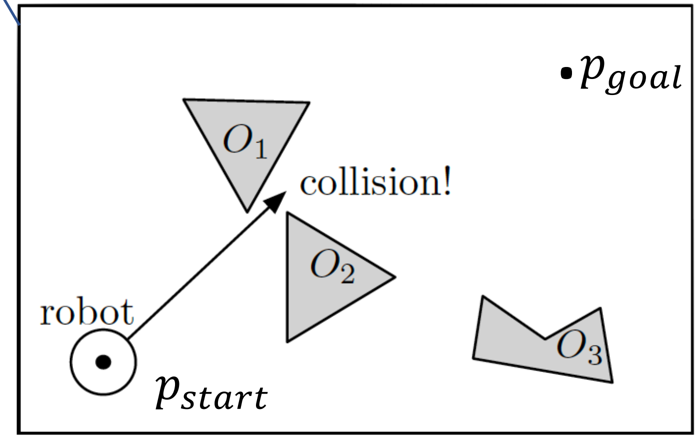
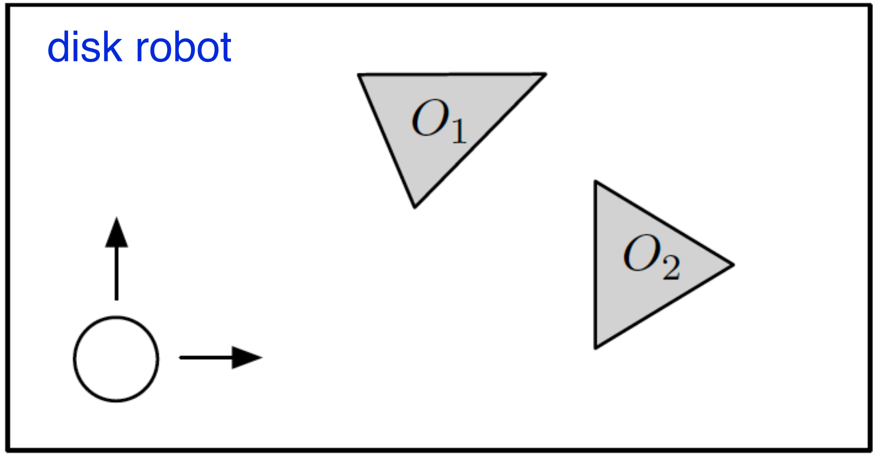
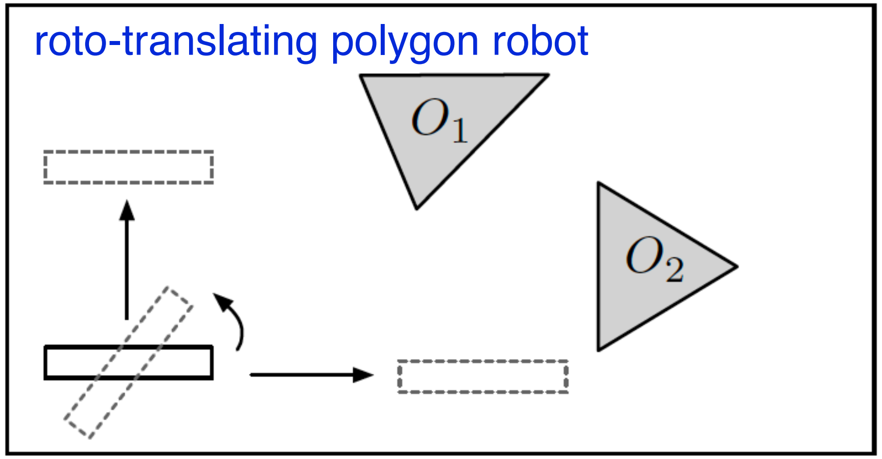
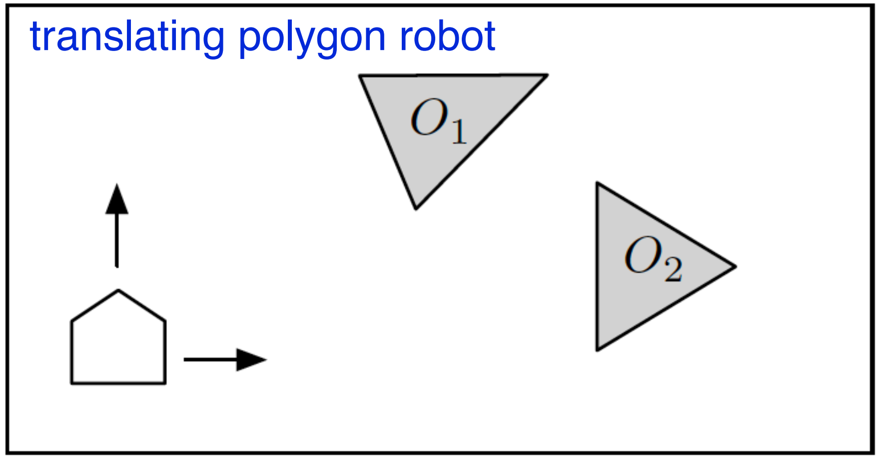
Example Robot Model Abstractions

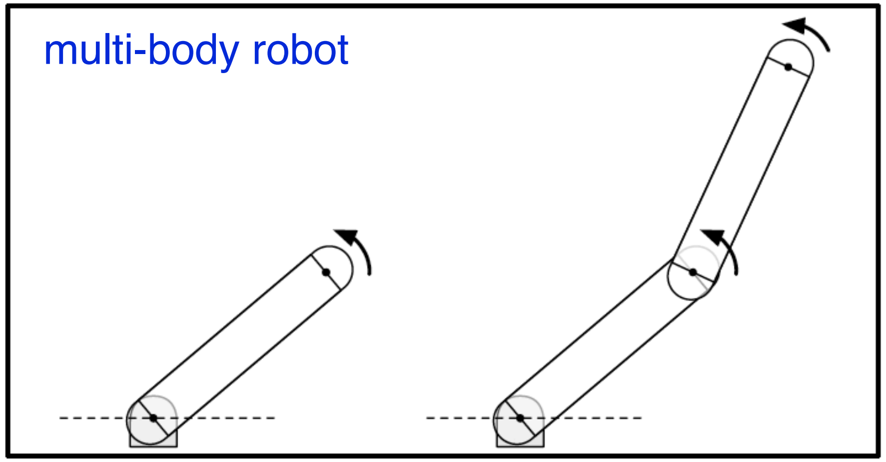
robots with a finite shape and size (a robot is composed of a single rigid body or multiple interconnected rigid bodies).











Motion Planning for Rigid Body Robots

To create motion plans for robots, we must be able to specify the position of the robot.

- We should ensure that no points on the robot collides with an obstacle.

we should be able to give a specification of the location of every point on the robot

- How much information is required to completely specify the position of every point on the robot?
- How should this information be represented?
- What are the mathematical properties of these representations?

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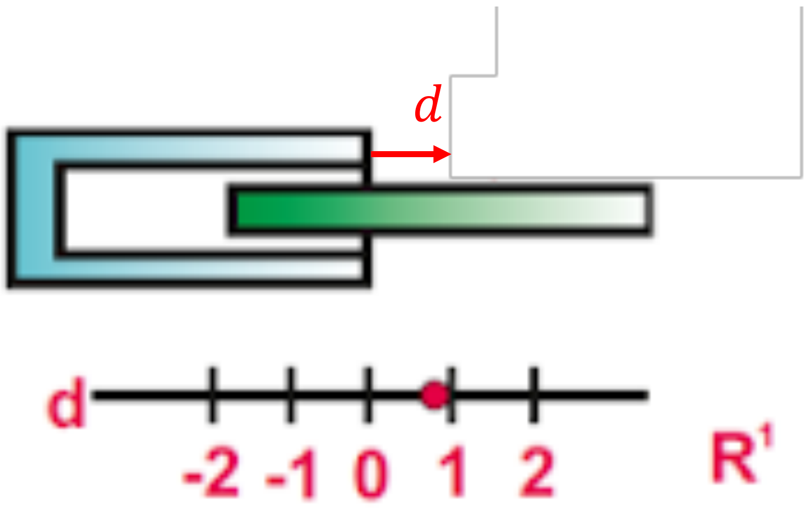
- How can obstacles in the robot world be taken into consideration while planning the path of a robot?

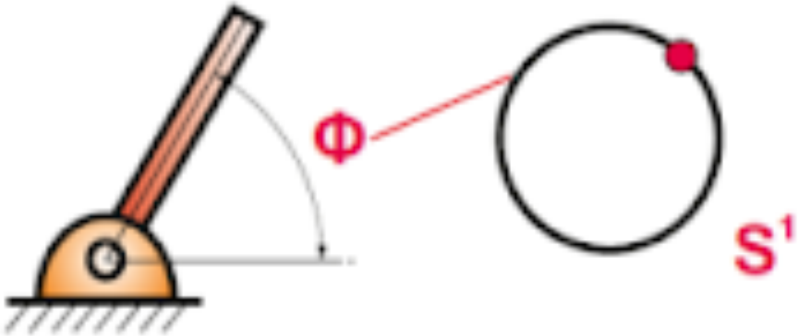
Configuration of a Robot

To specify the position of every point belonging to a rigid body:

the position of a specific point and the orientation of the rigid body, plus a representation of the shape of the rigid body (which does not vary with time as the body is rigid)

A minimal set of variables that describe the position and orientation of the specific point.





A configuration of a robot is a minimal set of variables that specifies the position and orientation of each rigid body composing the robot. The robot configuration is usually denoted by the letter q.

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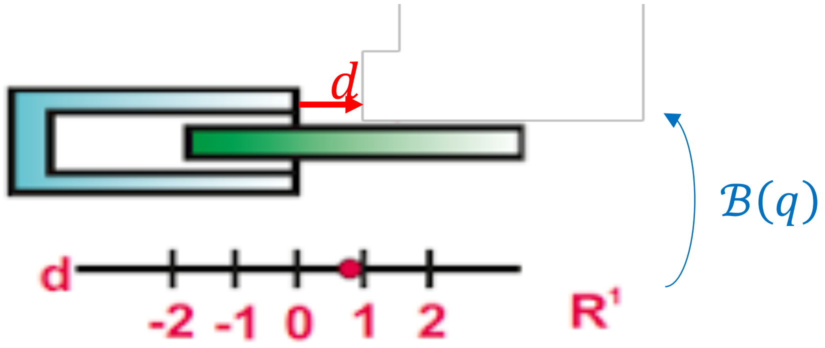
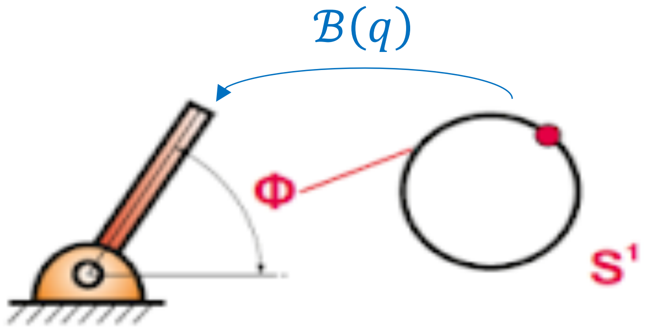


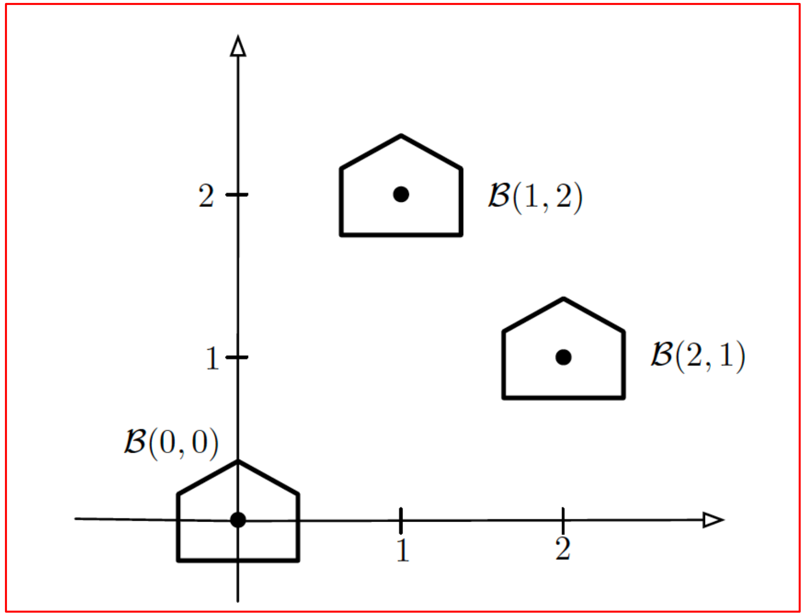


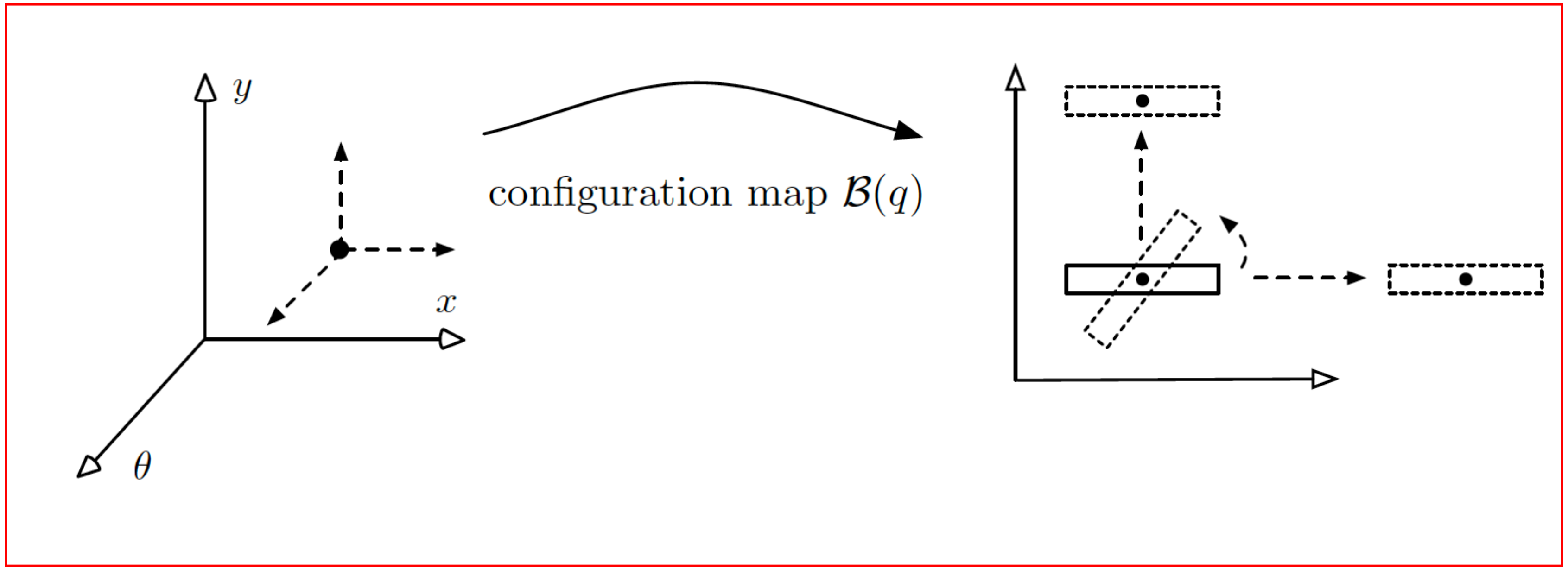
- The configuration space is the set of all possible configurations of a robot, denoted by the letter Q, so that $q \in Q$.

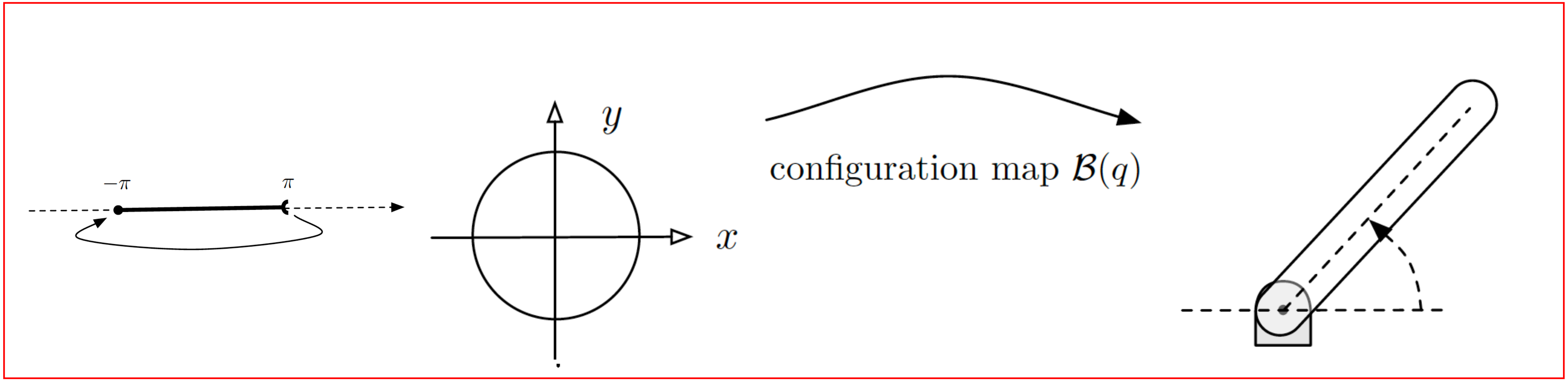
- The number of degrees of freedom of a robot is the dimension of the configuration space, i.e., the minimum number of variables required to fully specify the position and orientation of each rigid body belonging to the robot.

- The configuration map, and it maps each point $q \in Q$ to the set of all points $\mathcal{B}(q)$ of the workspace belonging to the robot.

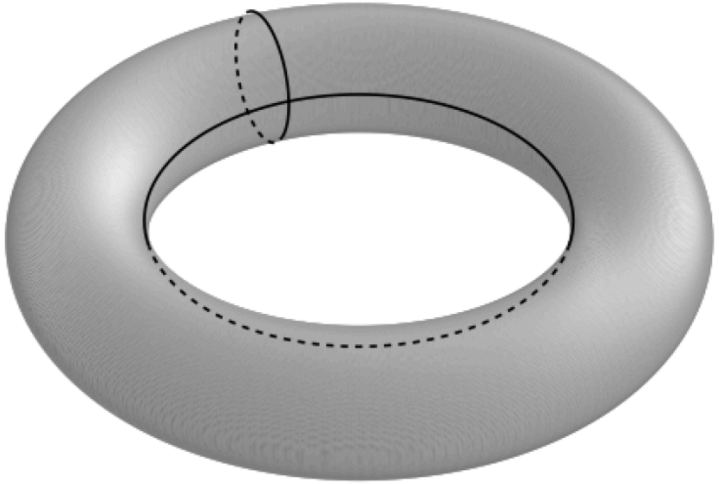
Configuration Space: Examples

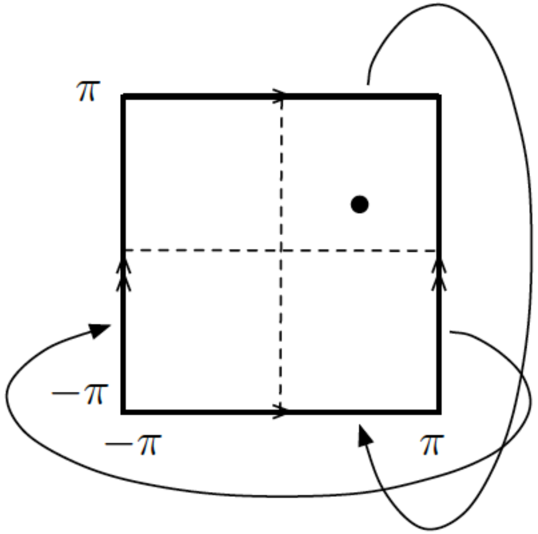




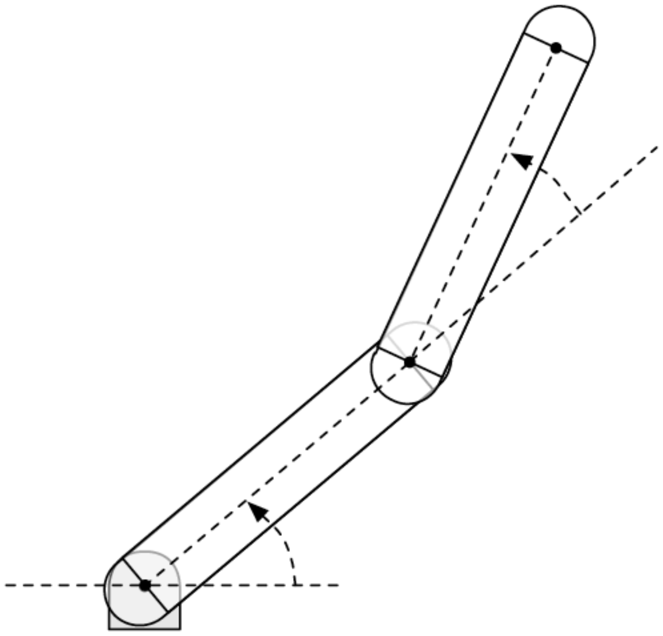


Configuration Space: Two-link Robot

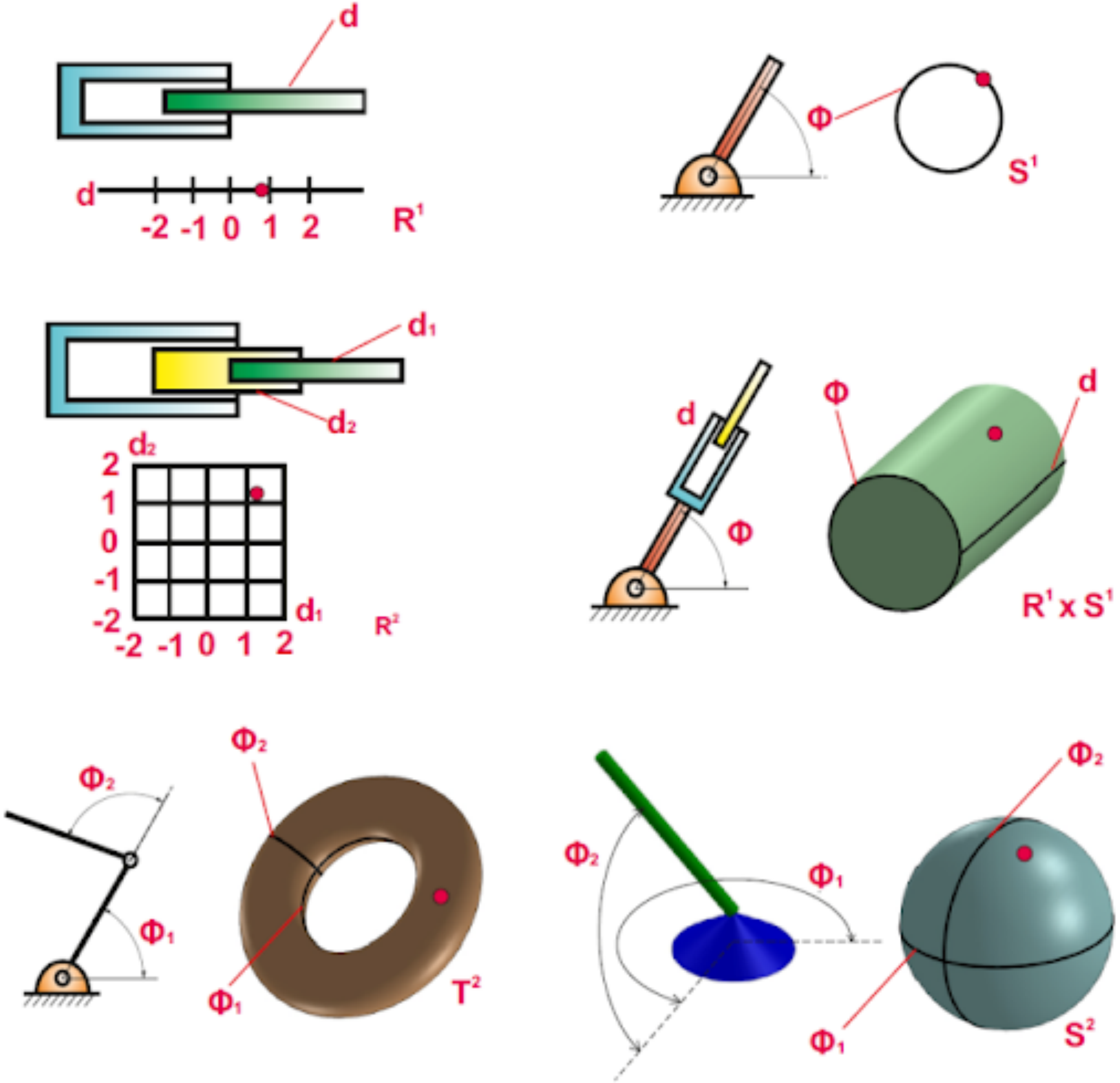




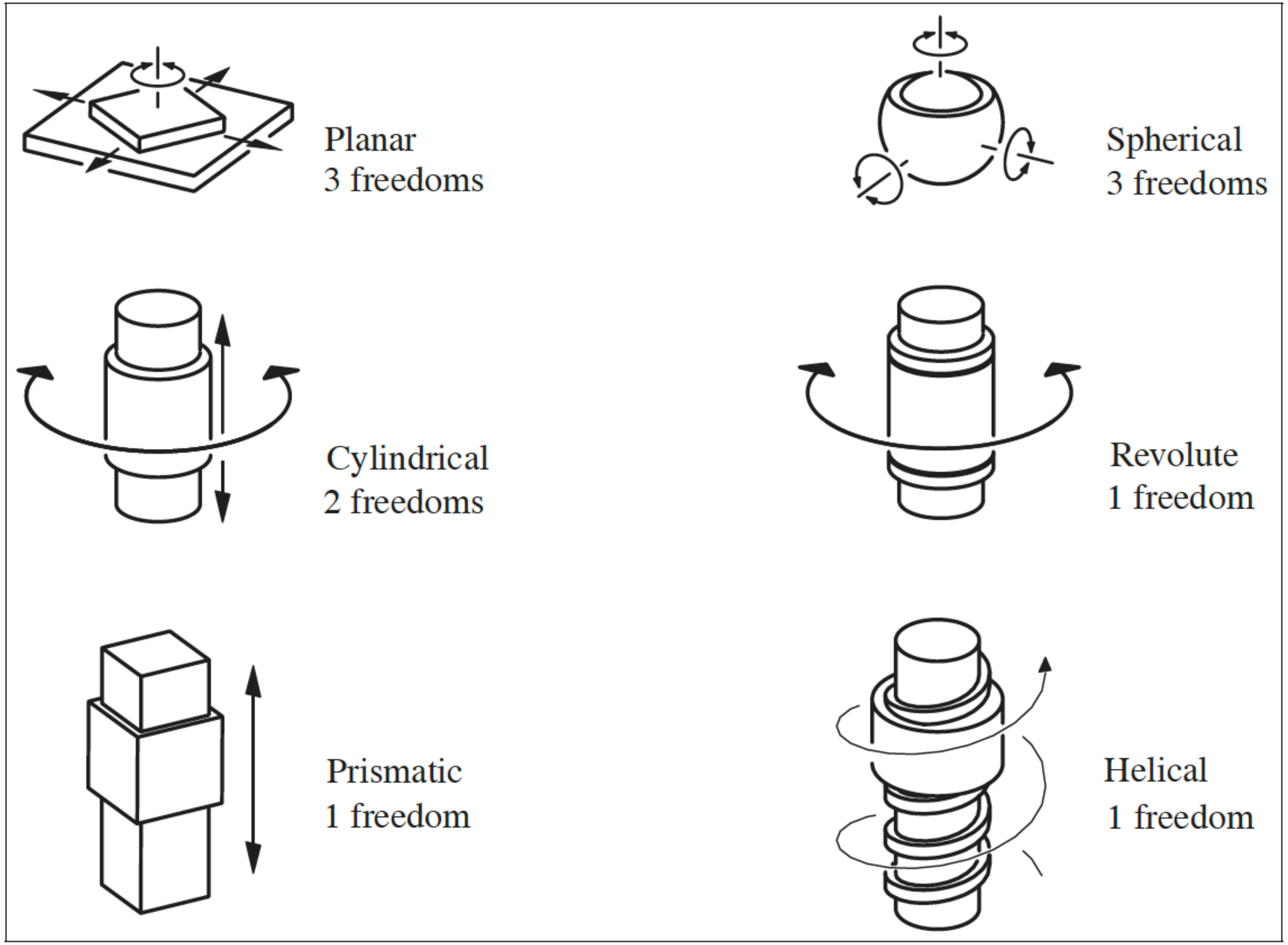
configuration map $\mathcal{B}(q)$



Configuration Space: More Examples



Joints and Their Degrees of Freedom



https://www:youtube:com/watch?v=5tRT5j3jfsE

Forward and Inverse Kinematic Maps

Motion planning for rigid body robots: given a motion planning problem in the workspace "move from point $p_{start} \in W$ to point $P_{goal} \in W$," we need to translate this specification into the configuration space, i.e., move from a configuration $q_{start} \in Q$ to a configuration $q_{goal} \in Q$.

With the help of forward and inverse kinematics maps we can transform motion planning problems from the workspace W to the configuration space Q.

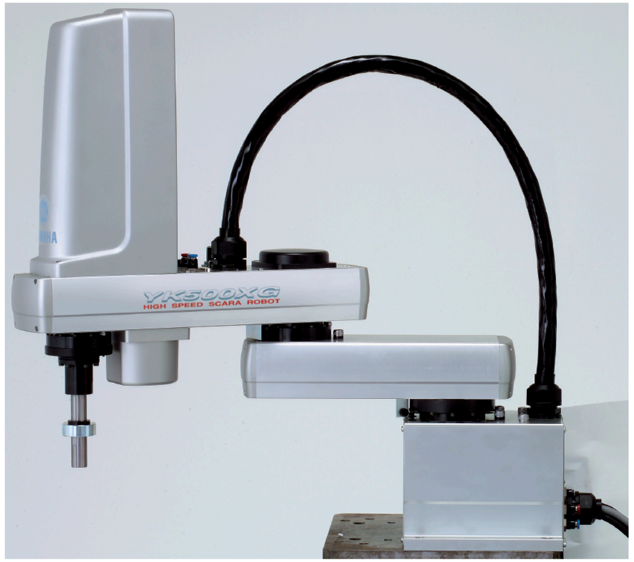


Figure 3.15: The Yamaha© YK500XG is a high-speed SCARA robot with two revolute joints and a vertical prismatic joint. Image courtesy of Yamaha Motor Co., Ltd, http:

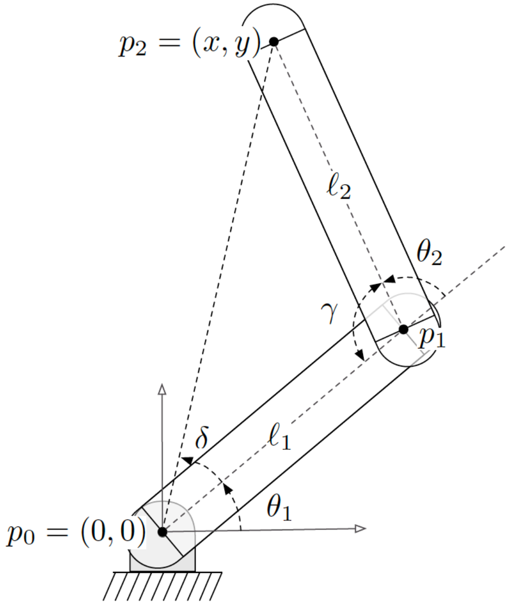
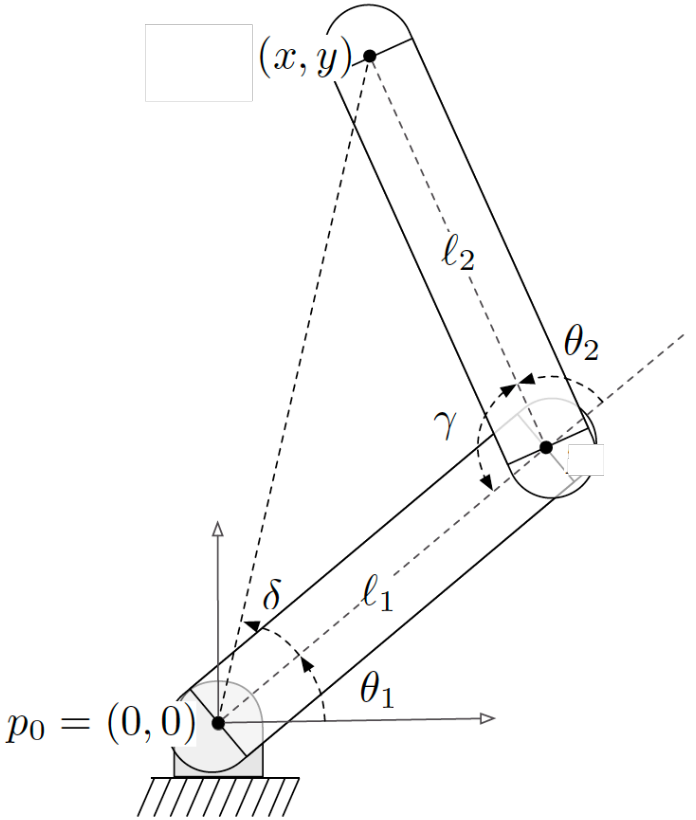


Figure 3.16: Vertical view of a SCARA robot, with end-effector location (x, y). In the triangle (p_0, p_1, p_2), define $\gamma \in [0, \pi]$ as the angle opposite the side $\overline{p_0 p_2}$.

The forward kinematics problem: compute (x, y) as a function of (θ_1, θ_2).

The inverse kinematics problem: compute(θ_1, θ_2) as a function of (x, y).

Forward Kinematics Map of the 2-link Robot



$$x = \ell_1 \cos(\theta_1) + \ell_2 \cos(\theta_1 + \theta_2),$$
$$y = \ell_1 \sin(\theta_1) + \ell_2 \sin(\theta_1 + \theta_2).$$

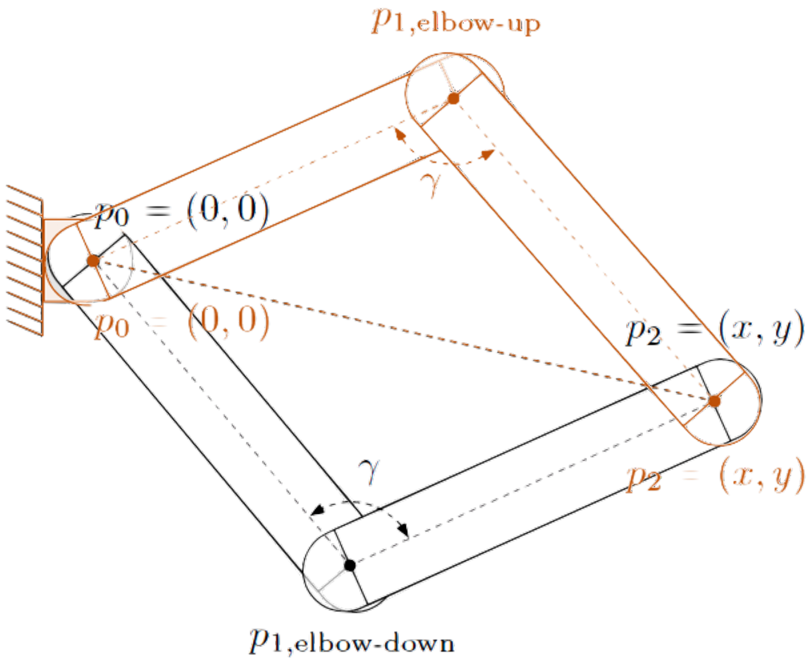
forward kinematics map

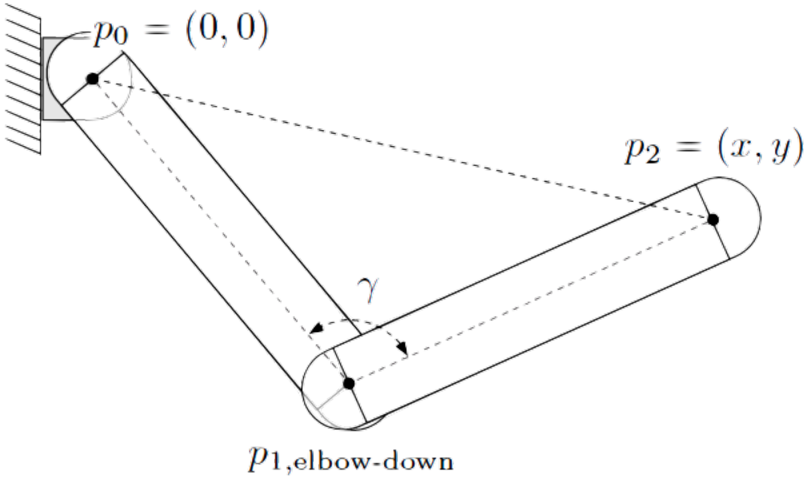
Inverse Kinematics Map of the 2-link Robot

Proposition (Inverse kinematics for 2-link robot) Consider the 2-link robot with configuration (θ_1, θ_2) and links length (ℓ_1, ℓ_2) shown below. Given a desired end-effector position (x, y) such that $|\ell_1 - \ell_2| < \sqrt{x^2 + y^2} \leq (\ell_1 + \ell_2)$, there exist two (possibly coincident) solutions for the joint angle θ_2 given by

$$\theta_{2,\text{elbow-down}} = \pi - \arccos\left(\frac{\ell_1^2 + \ell_2^2 - x^2 - y^2}{2\ell_1\ell_2}\right) \quad \in [0, \pi],$$

$$\theta_{2,\text{elbow-up}} = -\pi + \arccos\left(\frac{\ell_1^2 + \ell_2^2 - x^2 - y^2}{2\ell_1\ell_2}\right) \quad \in [-\pi, 0].$$





$$\cos(\gamma) = \frac{\ell_1^2 + \ell_2^2 - x^2 - y^2}{2\ell_1\ell_2}.$$

$$-1 \leq \frac{x^2 + y^2 - \ell_1^2 - \ell_2^2}{2\ell_1\ell_2} \leq 1 \iff -2\ell_1\ell_2 \leq x^2 + y^2 - \ell_1^2 - \ell_2^2 \leq 2\ell_1\ell_2$$

$$\iff (\ell_1 - \ell_2)^2 \leq x^2 + y^2 \leq (\ell_1 + \ell_2)^2$$

$$\iff |\ell_1 - \ell_2| \leq \sqrt{x^2 + y^2} \leq (\ell_1 + \ell_2).$$

Math Appendix: Solution to Basic Trigonometric Equalities

Given three numbers a, b, and c, we consider the following equations in the variables α, β and γ:

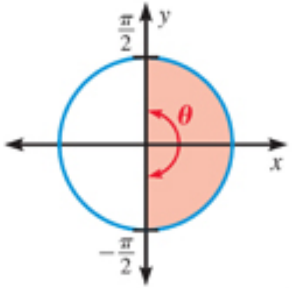
$$\sin(\alpha) = a, \quad \cos(\beta) = b, \quad \text{and} \quad \tan(\gamma) = c.$$

Inverse Trigonometric Functions

If $-1 \le a \le 1$, then the **inverse sine** of a is an angle θ, written $\theta = \sin^{-1} a$, where:

(1) $\sin\theta = a$

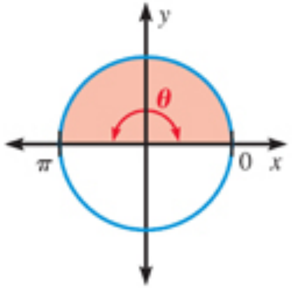
(2) $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$ (or $-90° \le \theta \le 90°$)



If $-1 \le a \le 1$, then the **inverse cosine** of a is an angle θ, written $\theta = \cos^{-1} a$, where:

(1) $\cos\theta = a$

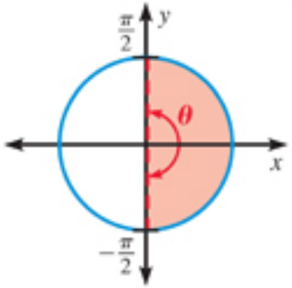
(2) $0 \le \theta \le \pi$ (or $0° \le \theta \le 180°$)



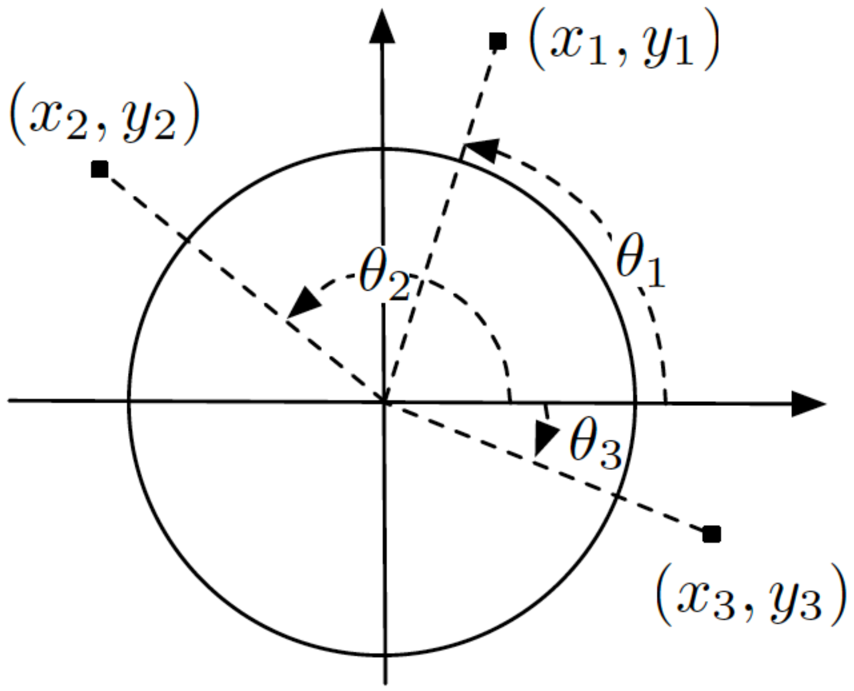
If a is any real number, then the **inverse tangent** of a is an angle θ, written $\theta = \tan^{-1} a$, where:

(1) $\tan\theta = a$

(2) $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ (or $-90° < \theta < 90°$)



Math appendix: Four-quadrant Arctangent Function with Two Arguments



$$\theta_1 = \text{atan}_2(y_1, x_1)$$
$$\theta_2 = \text{atan}_2(y_2, x_2)$$
$$\theta_3 = \text{atan}_2(y_3, x_3)$$

$$\text{atan2}(y, x) = \begin{cases} \arctan(\frac{y}{x}) & \text{if } x > 0 \\ \arctan(\frac{y}{x}) + \pi & \text{if } x < 0 \text{ and } y \geq 0 \\ \arctan(\frac{y}{x}) - \pi & \text{if } x < 0 \text{ and } y < 0 \\ +\frac{\pi}{2} & \text{if } x = 0 \text{ and } y > 0 \\ -\frac{\pi}{2} & \text{if } x = 0 \text{ and } y < 0 \\ \text{undefined} & \text{if } x = 0 \text{ and } y = 0 \end{cases}$$

Math Appendix: Alternative Solutions to Basic trigonometric Equalities

The four-quadrant arctangent function provides an alternative and sometimes easier way of computing solutions to

$$\sin(\alpha) = a, \quad \cos(\beta) = b, \quad \text{and} \quad \tan(\gamma) = c,$$

where we assume $|a| \leq 1$ and $|b| \leq 1$. Specifically, if $\sin(\alpha) = a$, then $\cos(\alpha) = \pm\sqrt{1-a^2}$ and therefore

$$\alpha_1 = \text{atan}_2(a, \sqrt{1-a^2}), \quad \text{and} \quad \alpha_2 = \text{atan}_2(a, -\sqrt{1-a^2}).$$

Similarly, if $\cos(\beta) = b$, then $\sin(\beta) = \pm\sqrt{1-b^2}$ and therefore

$$\beta_1 = \text{atan}_2(\sqrt{1-b^2}, b), \quad \text{and} \quad \beta_2 = \text{atan}_2(-\sqrt{1-b^2}, b).$$

Finally, $\tan(\gamma) = c$ is equivalent to

$$\gamma_1 = \text{atan}_2(c, 1), \quad \text{and} \quad \gamma_2 = \text{atan}_2(-c, -1).$$

References:

- F. Bullo and S. L. Smith. Lecture notes on robotic planning and kinematics
- H. Choset, K. Lynch, S. Hutchinson, G. Kantor, et al. Principles of Robot Motion, Theory, Algorithms, and Implementations.