HOMEWORK 6
MAE 206- OPTIMIZATION METHODS
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Problem 1. You have 10 ft of wire and you are going to cut it into two pieces. One piece you will bend into a circle, and one piece you will bend into a square. What ratio of the two pieces will maximize the combined area of the two shapes?

– Formulate this problem as an optimization problem.
– Solve the problems. Show your work.

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Problem 2. Consider the optimization problem below

$$\begin{aligned}&\text{P1 :minimize } f(x) = \frac{1}{2}(x_1^2 + x_2^2 + x_3^2), \quad \text{subject to,}\\ &x_1 + x_2 + x_3 = 3.\end{aligned}$$

– Recall the minimizer of this optimization problem from homework 5.
– Consider the penalty function form of this problem, i.e.,

$$\text{P2 : minimize } f_c(x) = \frac{1}{2}(x_1^2 + x_2^2 + x_3^2) + \frac{1}{2}c\,(x_1 + x_2 + x_3 - 3)^2.$$

find $(x_1^\star, x_2^\star, x_3^\star)$ in terms of c. For what values of c, the solutions of optimization P1 and P2 become equal. Plot $(c, x_1^\star(c))$, $(c, x_2^\star(c))$, and $(c, x_3^\star(c))$.

Problem 3. Find the maximizer of the problem below

$$\begin{aligned}&\text{maximize } f(x) = 14x - x^2 + 6y - y^2 + 7, \quad \text{subject to,}\\ &x + y \le 2,\\ &x + 2y \le 3.\end{aligned}$$