**HOMEWORK 5**
**MAE 206- OPTIMIZATION METHODS**
**INSTRUCTOR: PROF. SOLMAZ S. KIA**

**Problem 1**. Consider the optimization problem below

$$\text{P1 :minimize } f(x) = \frac{1}{2}(x_1^2 + x_2^2 + x_3^2), \quad \text{subject to,}$$
$$x_1 + x_2 + x_3 = 3.$$

- Find the minimizer of this optimization problem (using the first order and second order optimality conditions for constrained optimization).
- Use elimination method to eliminate the equality constraint and obtain the minimizer using the elimination method.

**Problem 2**. Determine the minimizer(s) of

$$\text{minimize } f(x) = x_1 + x_2^2 + x_2\, x_3 + 2x_3^2, \quad \text{subject to,}$$
$$\frac{1}{2}(x_1^2 + x_2^2 + x_3^2) = 1.$$

Use second order sufficiency condition to evaluate your candidate minimizer.

**Problem 3**. Show that the volume of the largest rectangular parallelepiped that can be inscribed in the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

is $\frac{8\,a\,b\,c}{3\sqrt{3}}$.

**Problem 4**. You have 10 ft of wire and you are going to cut it into two pieces. One piece you will bend into a circle, and one piece you will bend into a square. What ratio of the two pieces will maximize the combined area of the two shapes?

- Formulate this problem as an optimization problem.
- Solve the problems. Show your work.

**Problem 5**. (*This is an academic example*) Consider a deployment problem where the goal is to deploy two quadcopters in a 3D space such that the quadcopter C1 is on the plane that goes through $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 1)$ points, while quadcopter C2 is on xy plane. The objective is to place C1 and C2 on these planes such the distance between them is 2 meters and C1 is in closest possible distance to $P = (1, 1, 1)$ and C2 is in closest possible distance to $Q = (-1, -1, -1)$ (all the dimensions are in meters).

- Formulate this problem as an optimization problem.
- Solve the problems and find the optimal deployment position for the quadcopters. Show your work (you can use Matlab or any software you want to solve your set of the first order optimality equations).