Stien Coverage
A Distribution-Matching Multi-agent Deployment for Coverage

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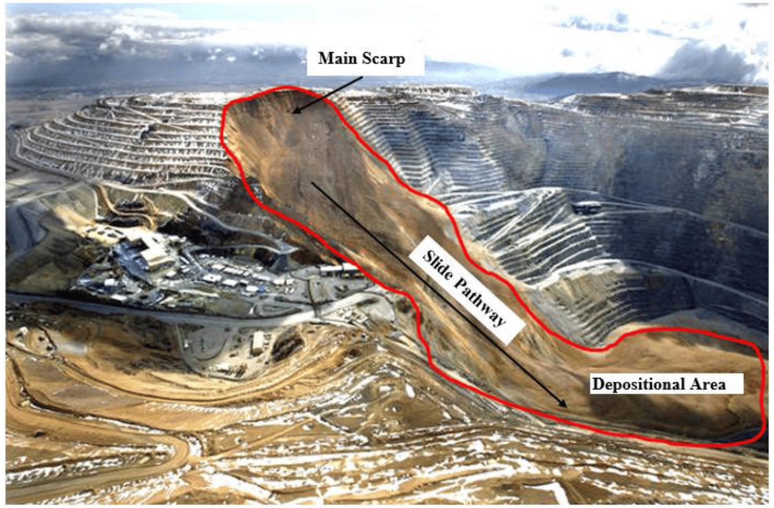
Multi-agent coverage problem



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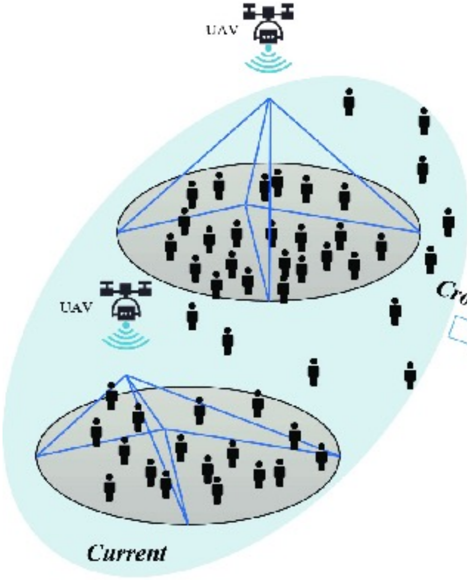

Multi-agent coverage problem: the challenge

Credit: https://www.flytbase.com/blog/drone-disaster-relief

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Credit: https://earth.org/how-do-oil-spills-affect-the-environment/

Vast area

Limited resources

In number Battery life

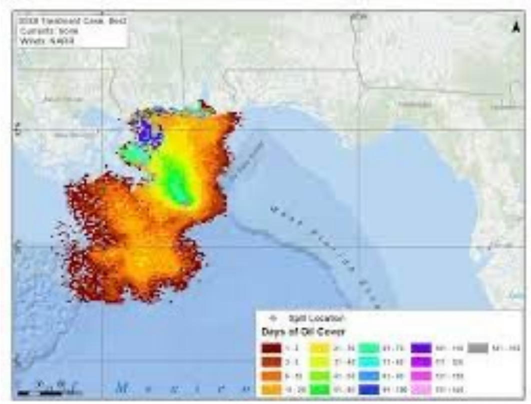
A ticking clock

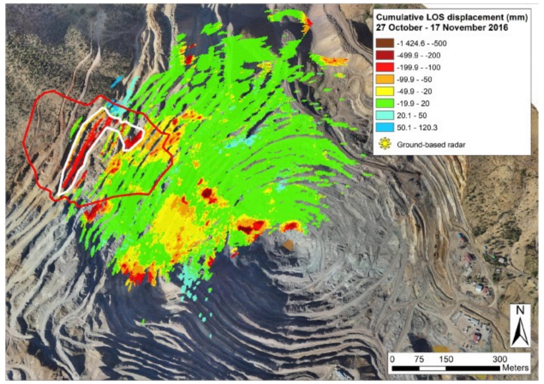
Objective: the best use of resources

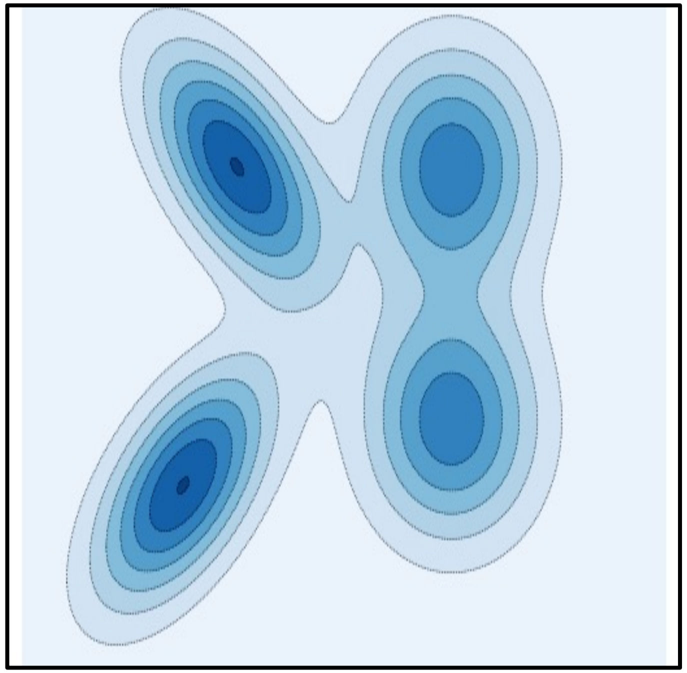
The coverage problem involves strategically placing a limited number of agents across the area of interest in such a way that a certain coverage measure is maximized.

How to approach the problem

Credit: https://earth.org/how-do-oil-spills-affect-the-environment/







The information at hand:

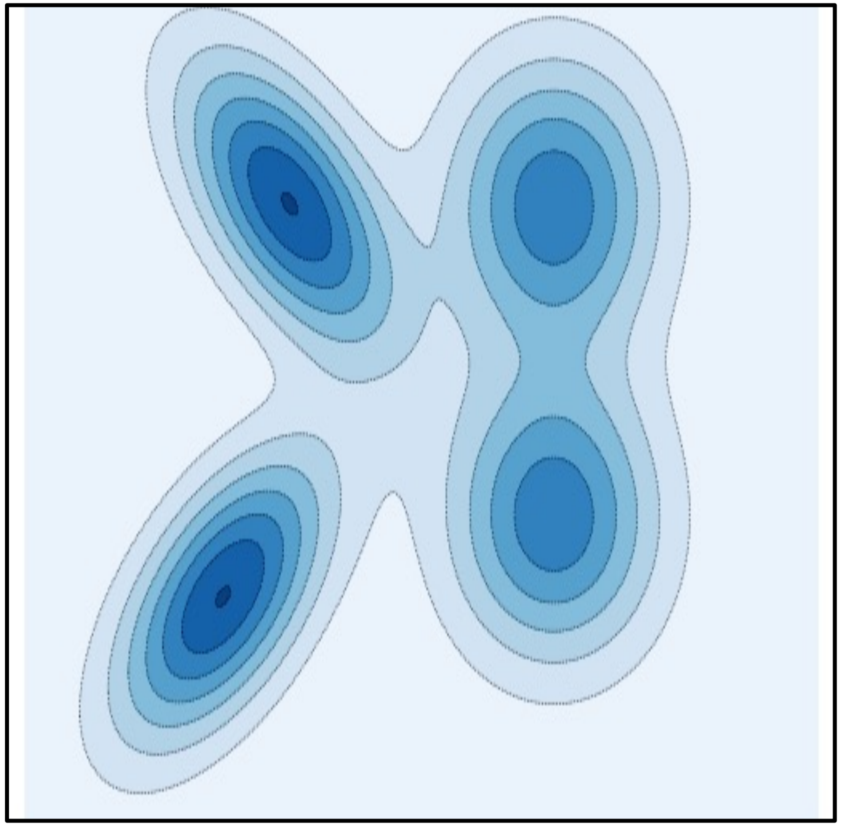
$p(x)$: distribution of the coverage event

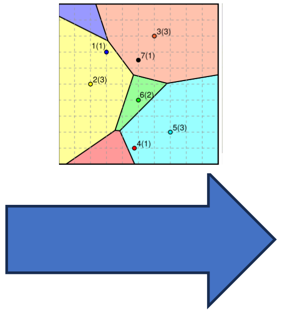
A rational solution to coverage

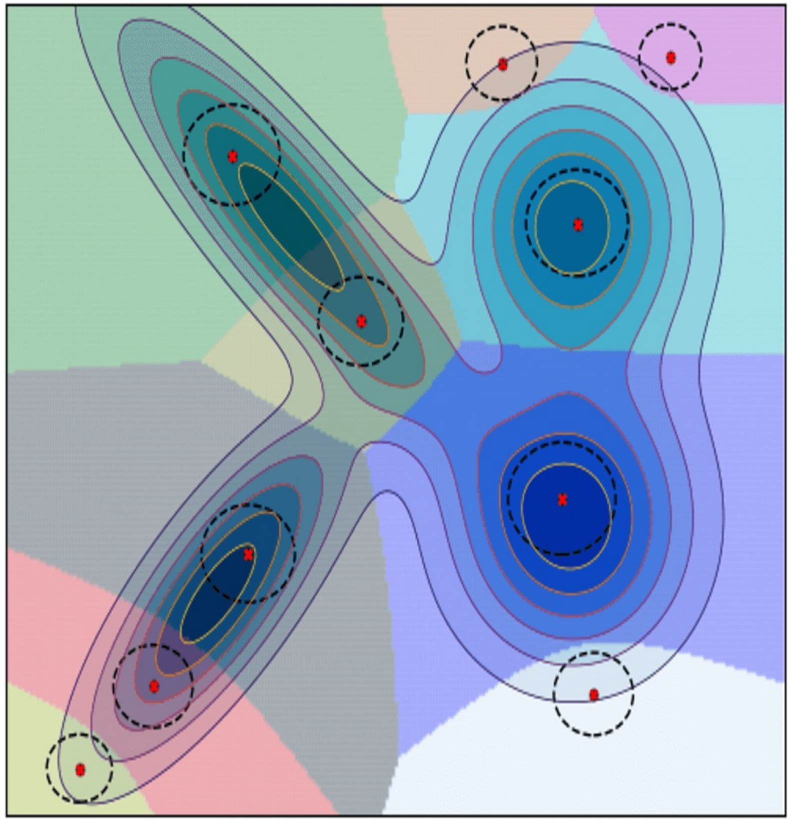
In coverage problems with a limited number of agents, the essence of the proposed solutions is

- divide the area into subregions and
- assign an agent to each one,

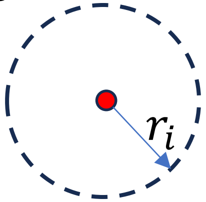
with the goal of maximizing a certain coverage measure.







Agent i

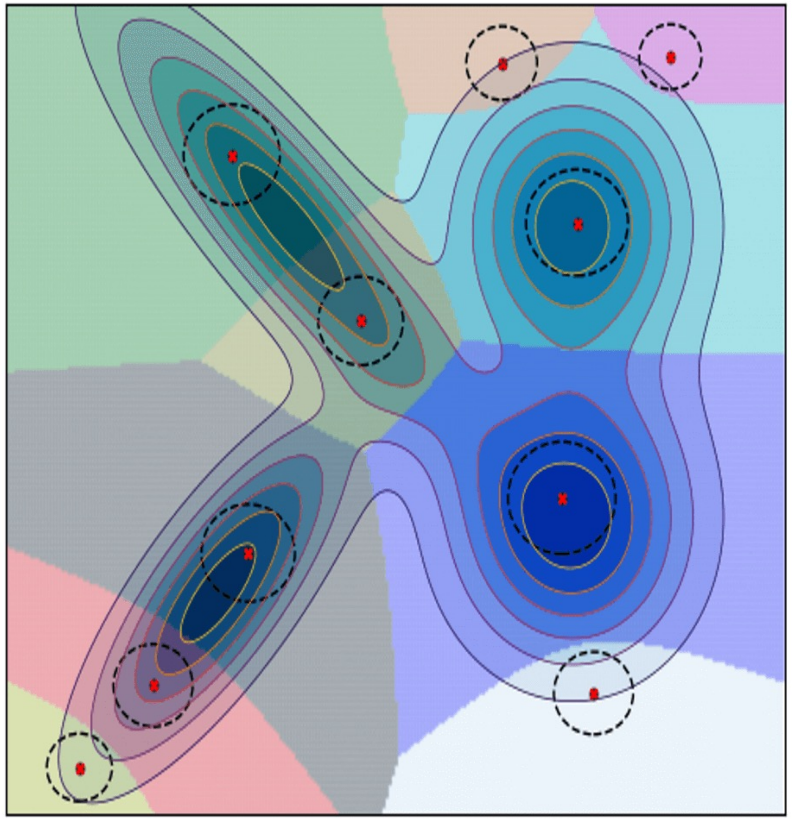


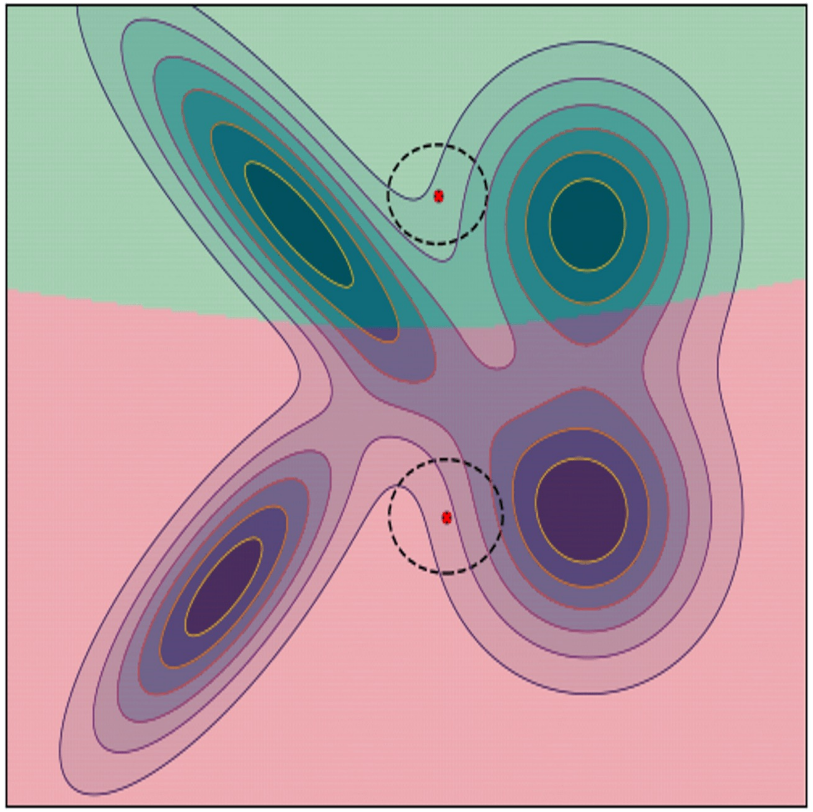
Coverage footprint

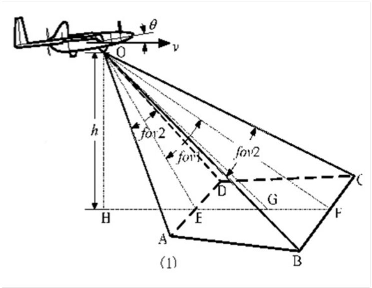
$$||x - x_i||^2 \leq r_i^2$$

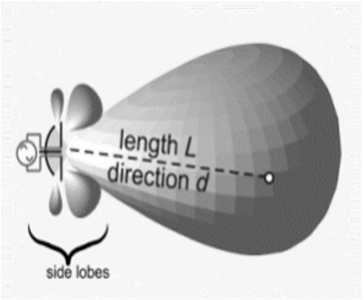
$$\{x_i\}_{i=1}^{n} \leftarrow \text{minimize} \sum_{i=1}^{n} \int \left(||x - x_i||^2 - r_i^2\right) p(x) dx$$

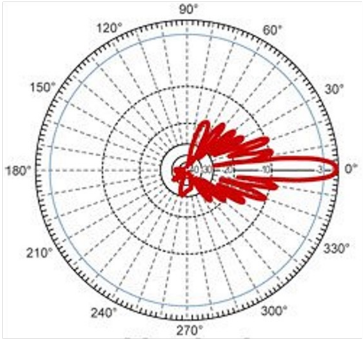
Why alternatives to Voronoi partitioning?

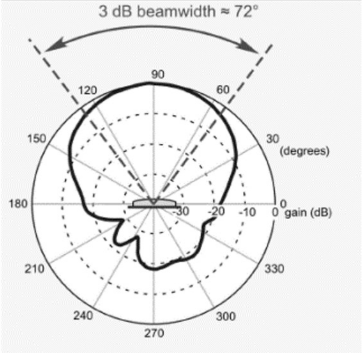




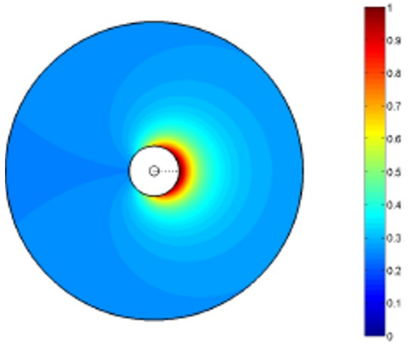




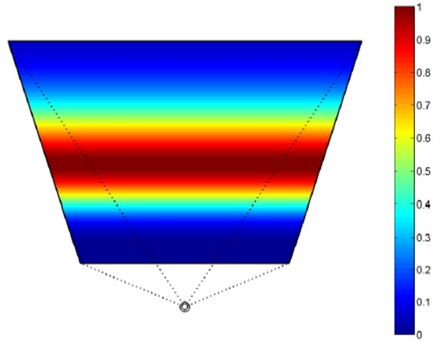




$$\{x_i\}_{i=1}^{n} \leftarrow \text{minimize} \sum_{i=1}^{n} \int (||x - x_i||^2 - r_i^2) p(x) dx$$



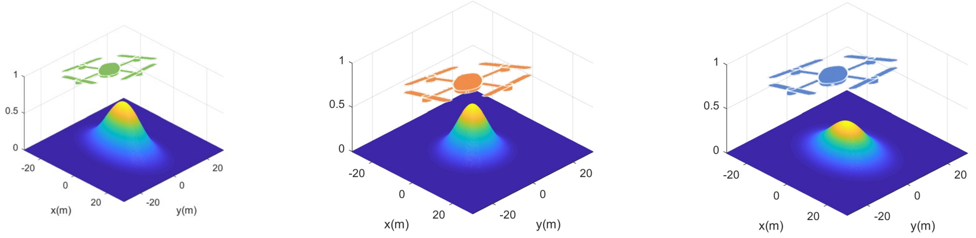
Acoustic sensor



Camera

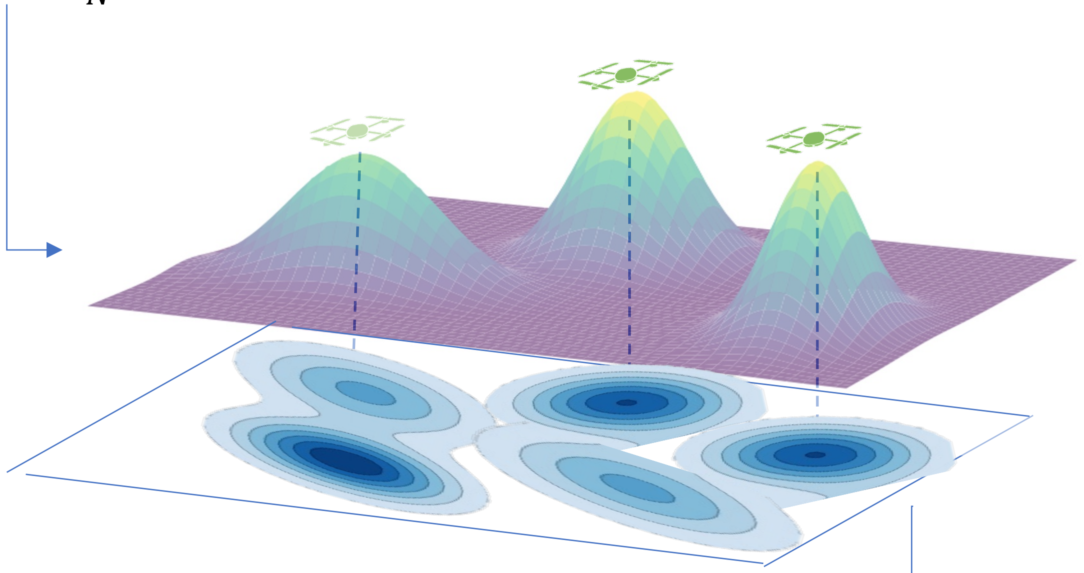
Distribution-matching deployment

Every agent has
- a quality-of-service distribution $s_i(x|x_i, \theta_i)$, and
- An effective coverage footprint $\mathcal{C}_i(x|x_i, \theta_i)$



Collective quality of service of the agents

$q(x) = \frac{1}{N}\sum_{i=1}^{N} s_i(x|x_i, \theta_i)$:

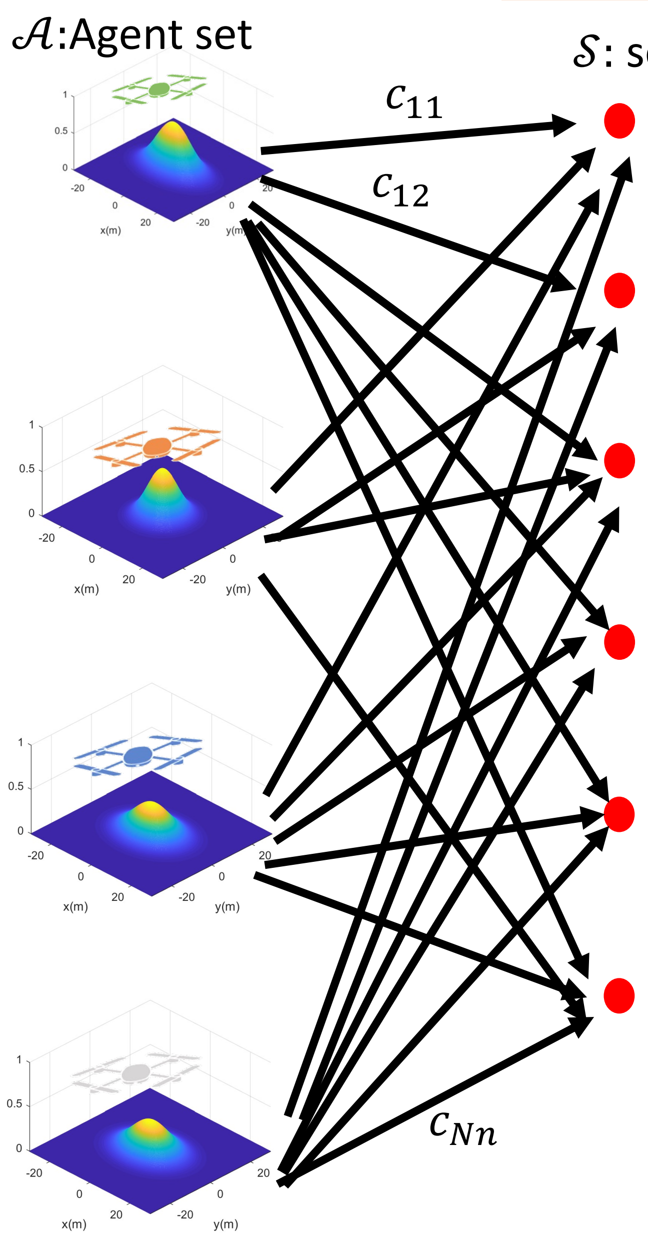


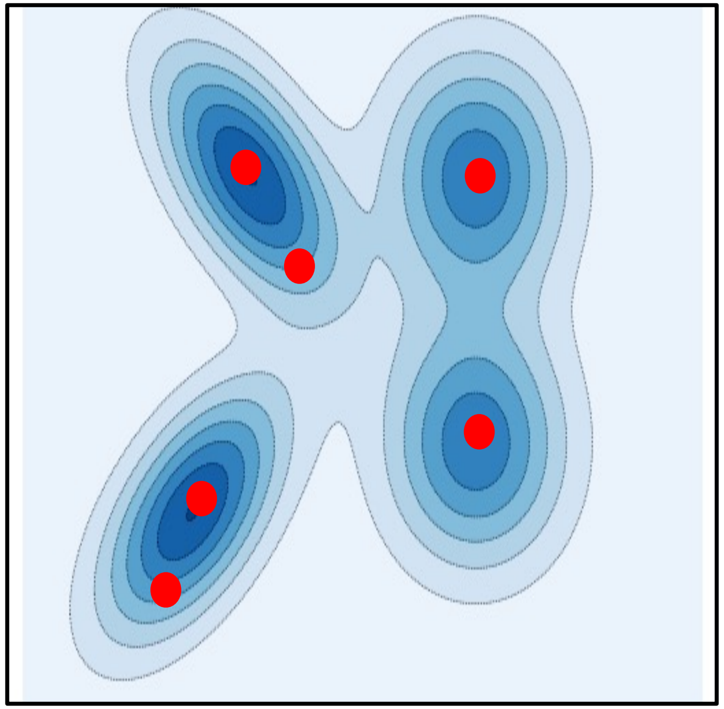
$p(x)$: coverage event distribution

Distribution–Matching Deployment

$$(x_i^\star, \theta_i^\star)_{i=1}^N = \operatorname{argmin} \mathcal{KL}[q(x)||p(x)]$$

Distribution-matching deployment: proposed solution





$$C^{\star}_{i,j} = \min_{\theta_i \in \Theta} \left\{ \mathcal{KL}\left(s(x|x_j,\theta_i) || p(x) \right) \; for \; x \in \mathcal{C}_i(x|x_j,\theta_i) \right\}$$

Optimal Agent Assignment

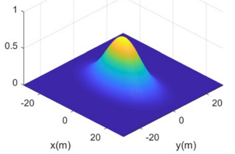
$$\mathbf{Z}^{\star} = \arg\min \sum_{j\in\mathcal{S}} \sum_{i\in\mathcal{A}} Z_{i,j} C^{\star}_{i,j},$$
$$Z_{i,j} \in \{0,1\}, \quad i \in \mathcal{A}, \; j \in \mathcal{S},$$
$$\sum_{i\in\mathcal{A}} Z_{i,j} = 1, \; \forall i \in \mathcal{A},$$
$$\sum_{j\in\mathcal{S}} Z_{i,j} \leq 1, \; \forall j \in \mathcal{S}.$$

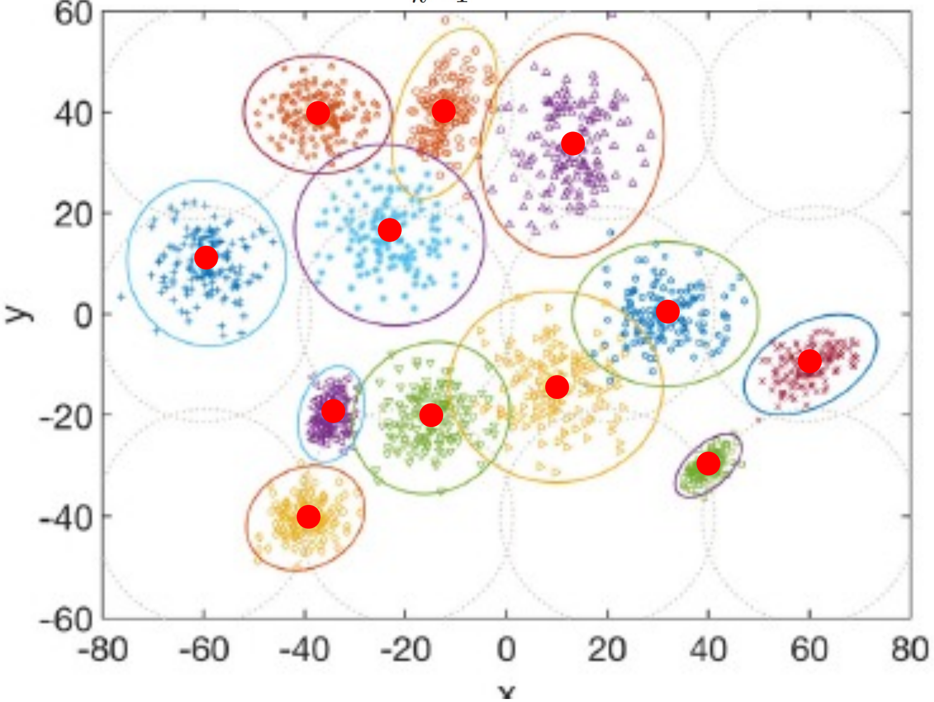
Research question: how to determine the PoIs?

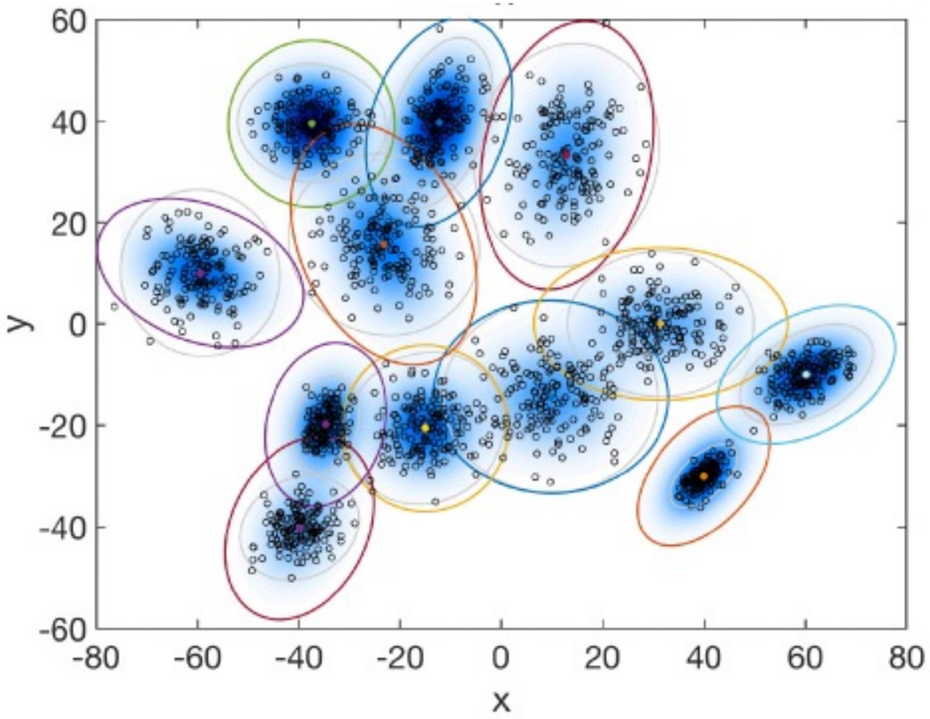
Distribution-matching deployment: a GMM clustering approach

$$p^i(\mathbf{x}) = \sum_{k=1}^{N_s} \pi_k^i \mathcal{N}(\mathbf{x}|\boldsymbol{\mu}_k^i, \boldsymbol{\Sigma}_k^i)$$

$$s_i(x|x_i,\theta_i) = \mathcal{N}\left(x|x_i, \begin{bmatrix} (\sigma_x^i)^2 & 0 \\ 0 & (\sigma_y^i)^2 \end{bmatrix}, \theta_i\right)$$





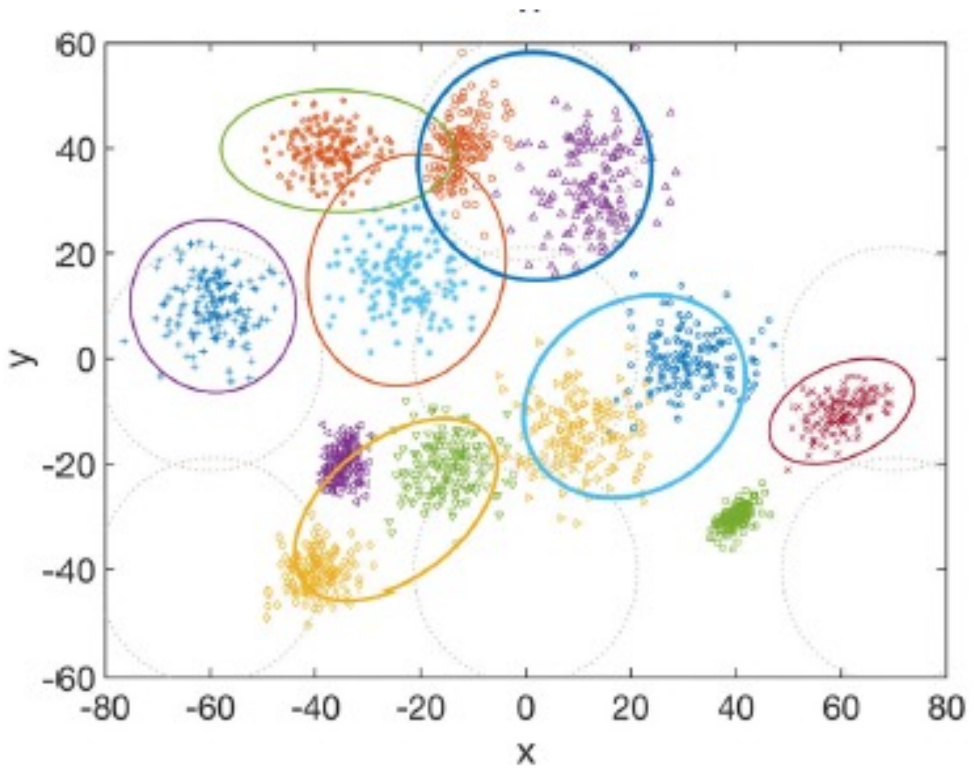


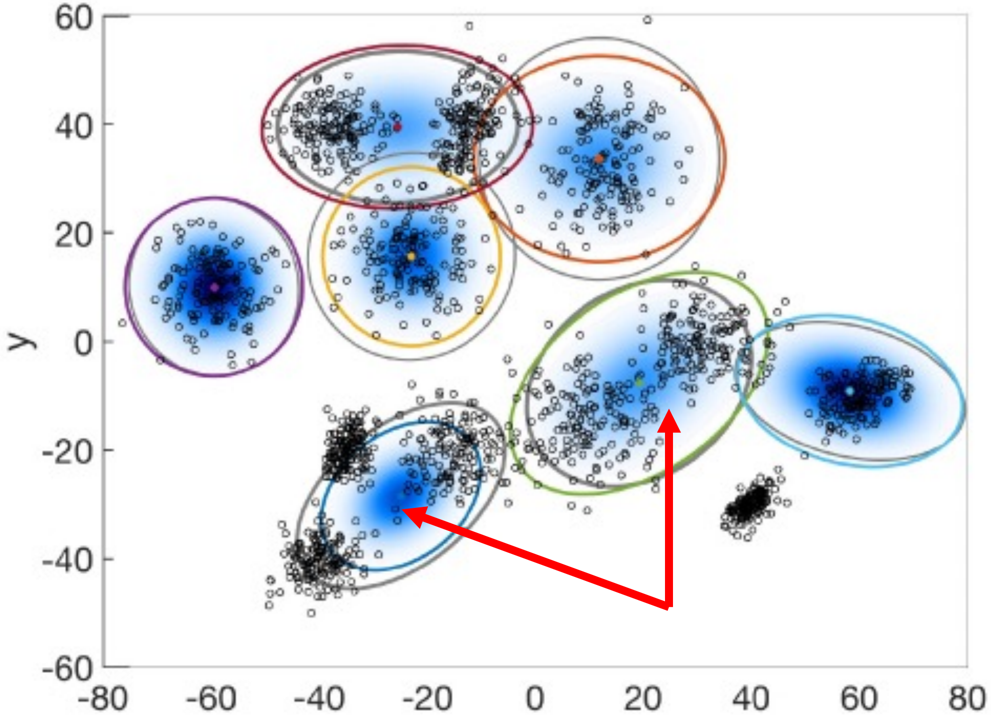
PoIs: center of basis of GMM

$$C_{i,j}^{\star} = \pi_k^i \left(\ln \frac{\pi_k^i}{\omega_s^i} + \frac{1}{2} \left(\ln \frac{\sigma_x^i \sigma_y^i}{\sigma_{k,x}^i \, \sigma_{k,y}^i} + \frac{\sigma_{k,x}^i \sigma_y^i + \sigma_{k,y}^i \sigma_x^i}{\sigma_x^i \sigma_y^i} - 2 \right) \right)$$

Y. Chung and S. S. Kia, “A distributed service-matching coverage via heterogeneous agents,” IEEE Robotics and Automation Letters, vol. 7, no. 2, pp. 4400–4407, 2022.

Distribution-matching deployment: a GMM clustering approach



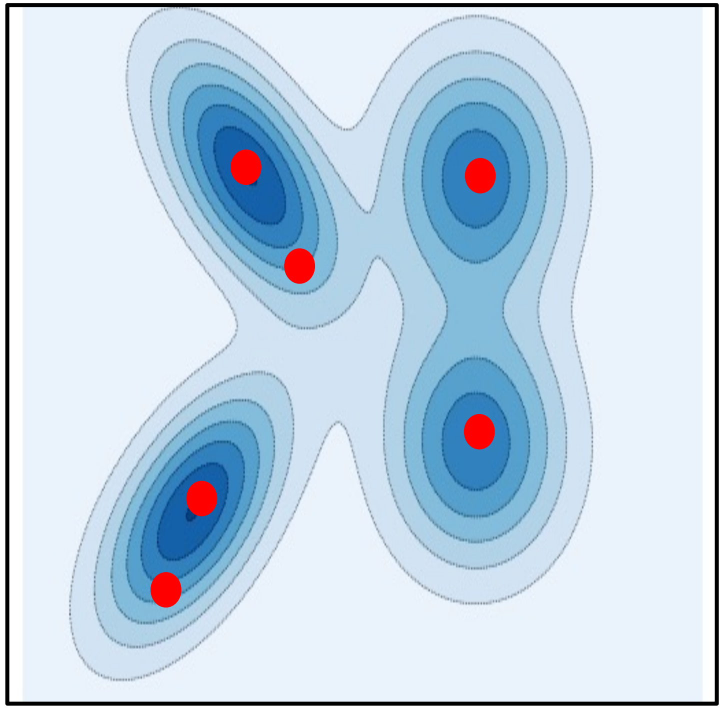


- Clustering is sensitive to initialization
- Does not necessarily do a good job in clustering for lower number of robots than real number of clusters
- Does not handle the overlapping area well

Extracting points of interests using Stien varational gradient descent

Use statistical methods to find samples from the distribution $p(x)$:

- Markov Chain Monte Carlo (MCMC)
- **Stein variational Gradient Descent (SVGD)**
  [Q. Liu, J. Lee, and M. Jordan, ICML , 2016]



D. Ghimire and S. S. Kia, "Optimal multi-sensor deployment via sample-based quality-of-service distribution matching," IEEE Robotics and Automation Letters (submitted) 2023, available at https://arxiv.org/abs/2312.07001

Extracting points of interests using Stien varational gradient descent

Algorithm 1 Stein Variational Gradient Descent

Input: The score function $\nabla_x \log p(x)$.

Goal: A set of particles $\{x_i\}_{i=1}^n$ that approximates $p(x)$.

Initialize a set of particles $\{x_i^{(0)}\}_{i=1}^n$; choose a positive definite kernel $k(x, x')$ and step-size.

For iteration l **do**

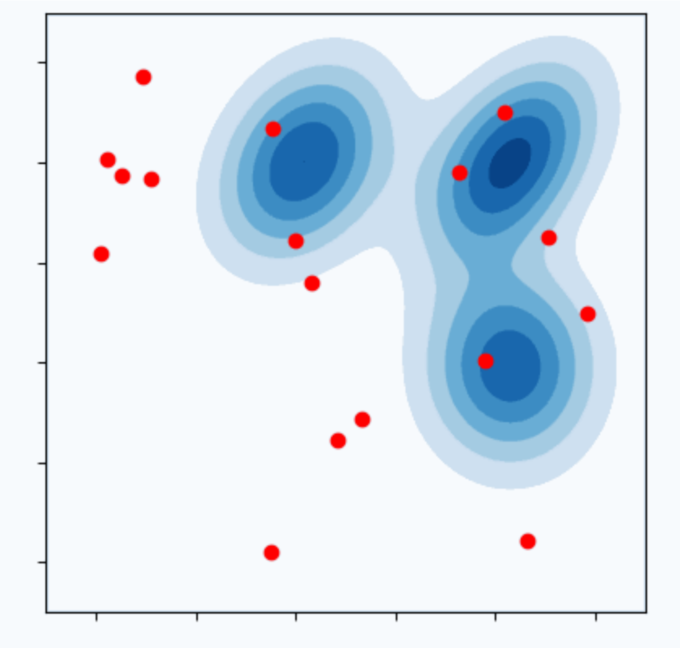
$$x_i^{(\ell+1)} \leftarrow x_i^{(\ell)} + \phi^\star(x_i^{(\ell)}) \quad \forall i \in \{1, \dots, n\}$$

where,

$$\phi^\star(x) = \frac{1}{n}\sum_{j=1}^n \left[\nabla \log p(x_j) k(x_j, x) + \nabla_{x_j} k(x_j, x)\right].$$

a weighted gradient of the log $p(x)$

known as the repulsive force, intuitively pushes particles apart when they approach each other closely, preventing them from collapsing into a single mode.

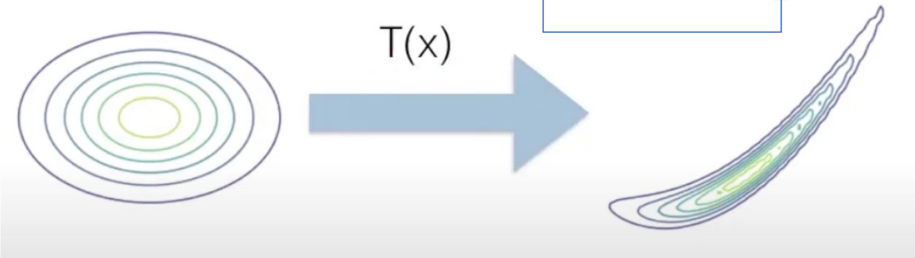


SVGD: Inference through deterministic map

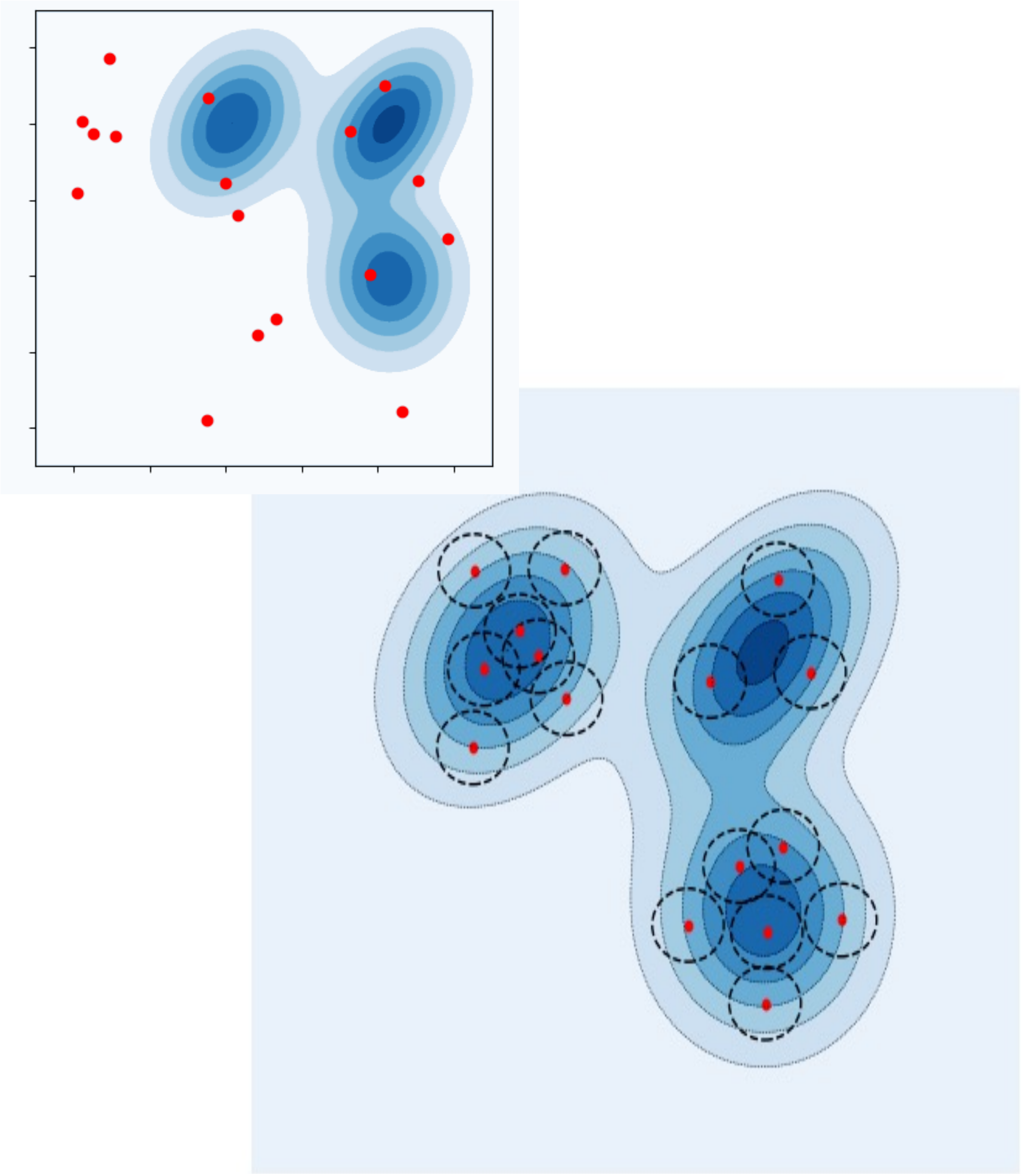
$$x \leftarrow T(x) = x + \epsilon\phi(x)$$

Perturbation direction

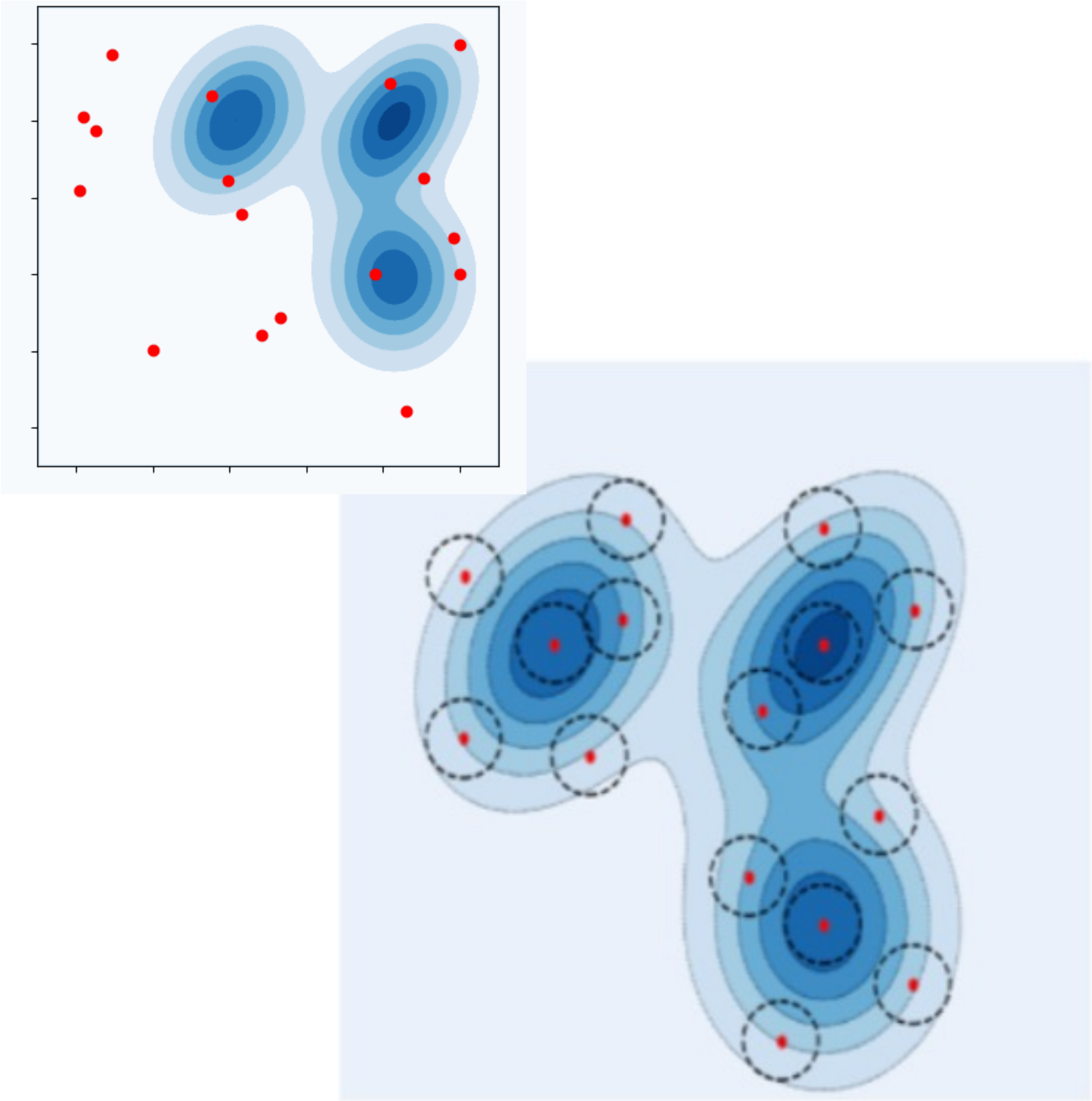
Step size



Extracting points of interests using Stien varational gradient descent: how to avoid overlaps

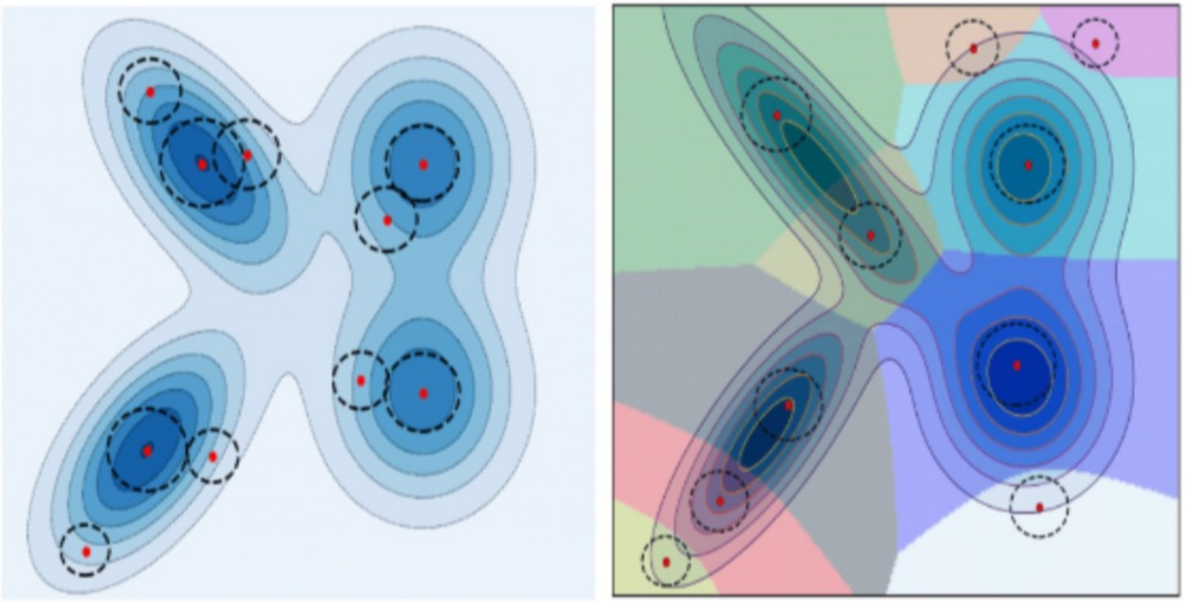


Trivial implementation of the SVGD

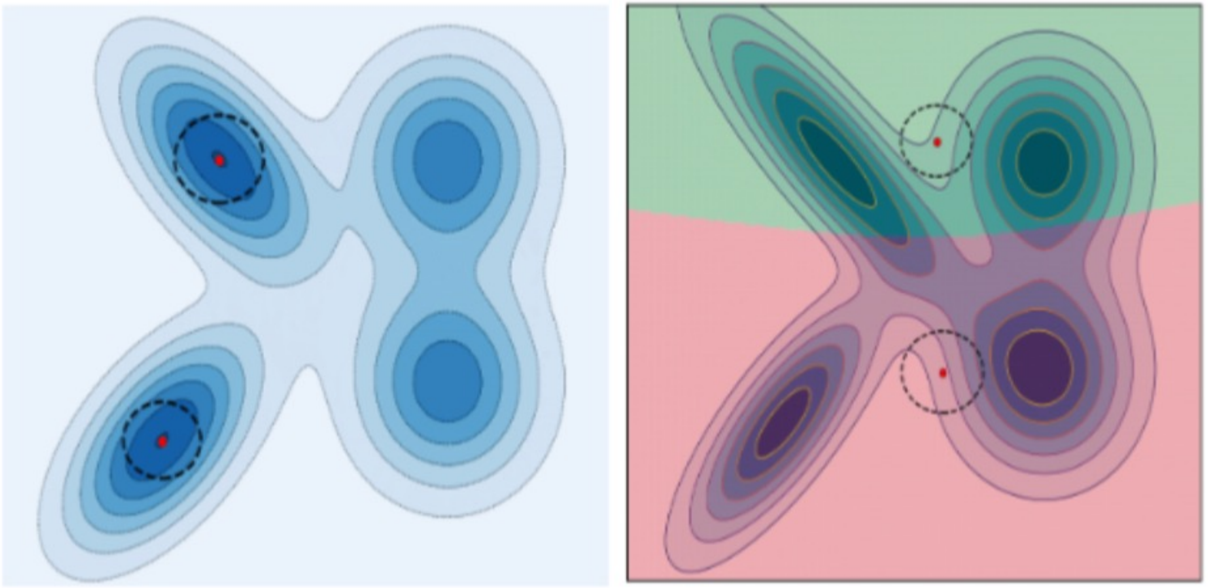


Through proper design of the repulsive force of SVGD

Distribution-matching deployment: proposed solution

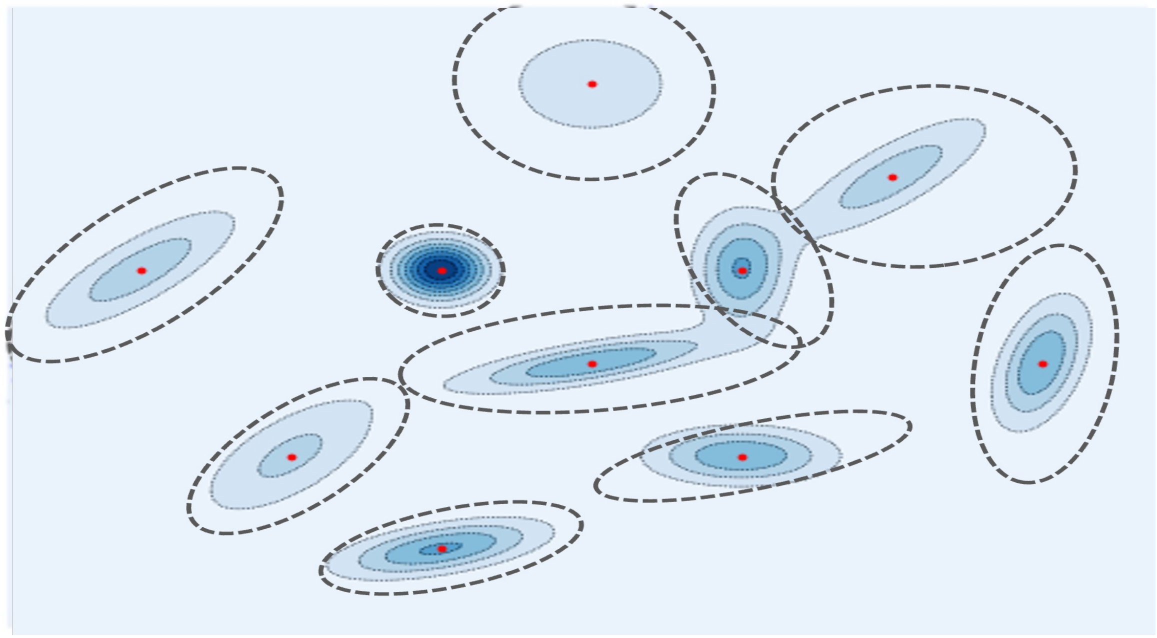


(a) Sensor deployment for heterogeneous sensors: (left) Stein Coverage, (right) Voronoi partitioning using power diagrams.

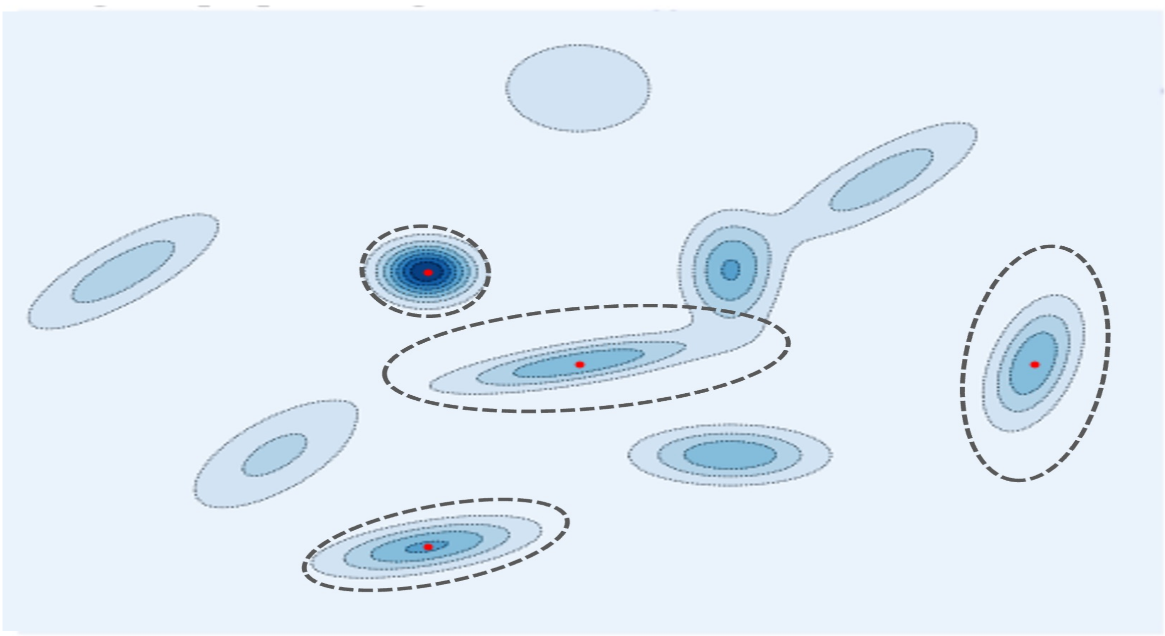


(b) Comparison of sensor deployment:(left) proposed Stein Coverage, (right) Voronoi partitioning using power diagrams, when the number of sensors is less than the number of clusters.

Distribution-matching deployment: proposed solution



(a) Sensor deployment for anisotropic heterogeneous sensors. The level sets represent the multimodal distribution of the target points (blue dots) and the ellipsoid encircling the level sets represents the configuration of anisotropic footprint calculated using the proposed algorithm.

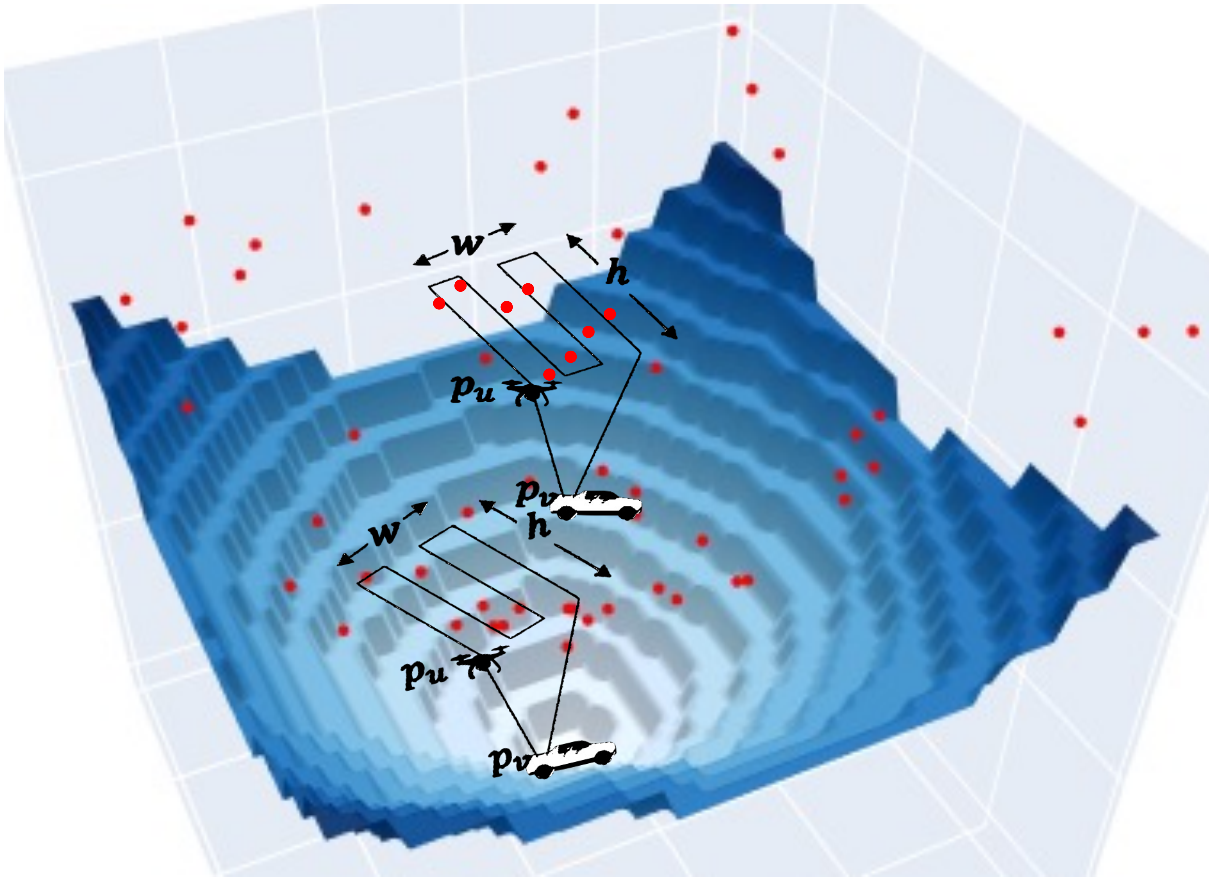


(b) Sensor deployment for heterogeneous anisotropic sensors when the number of sensors is less than the number of clusters.

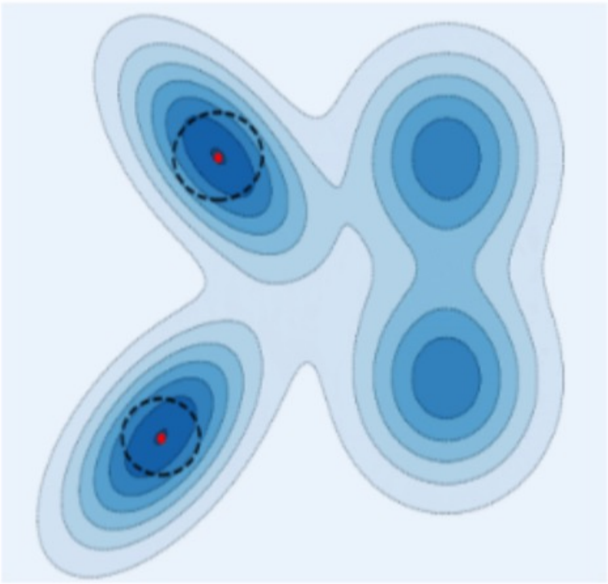
Area monitoring as a deployment problem

Monitoring problems can be cast as deployment problem:

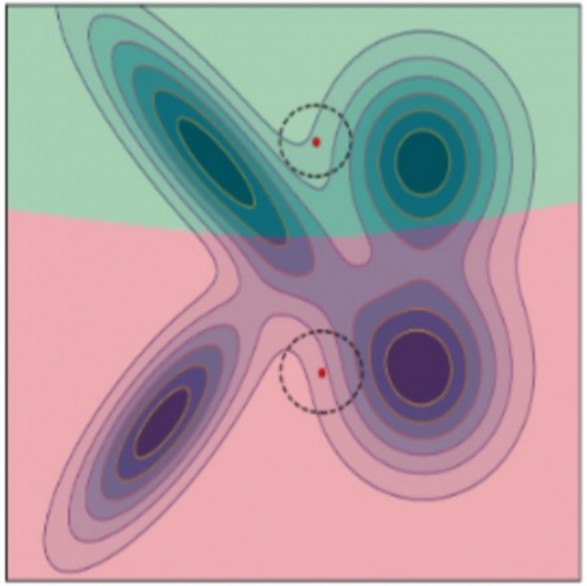
- Using predefined sweep patterns
- Footprint is defined by battery life



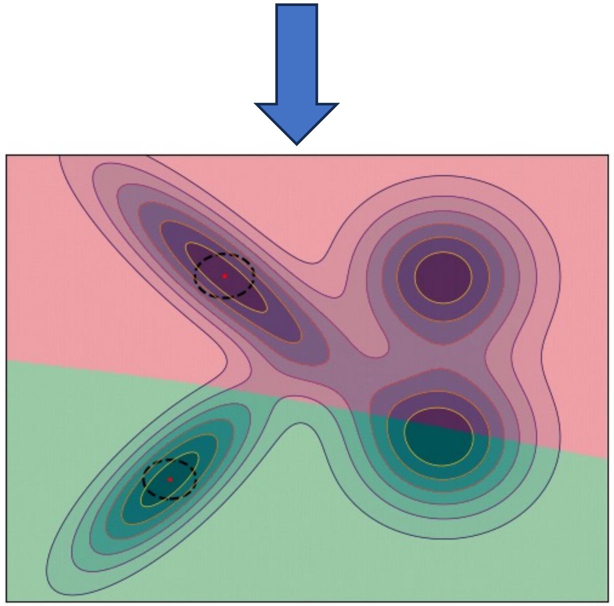
Open surface minefield monitoring



Stien Coverage



Voronoi-based solution



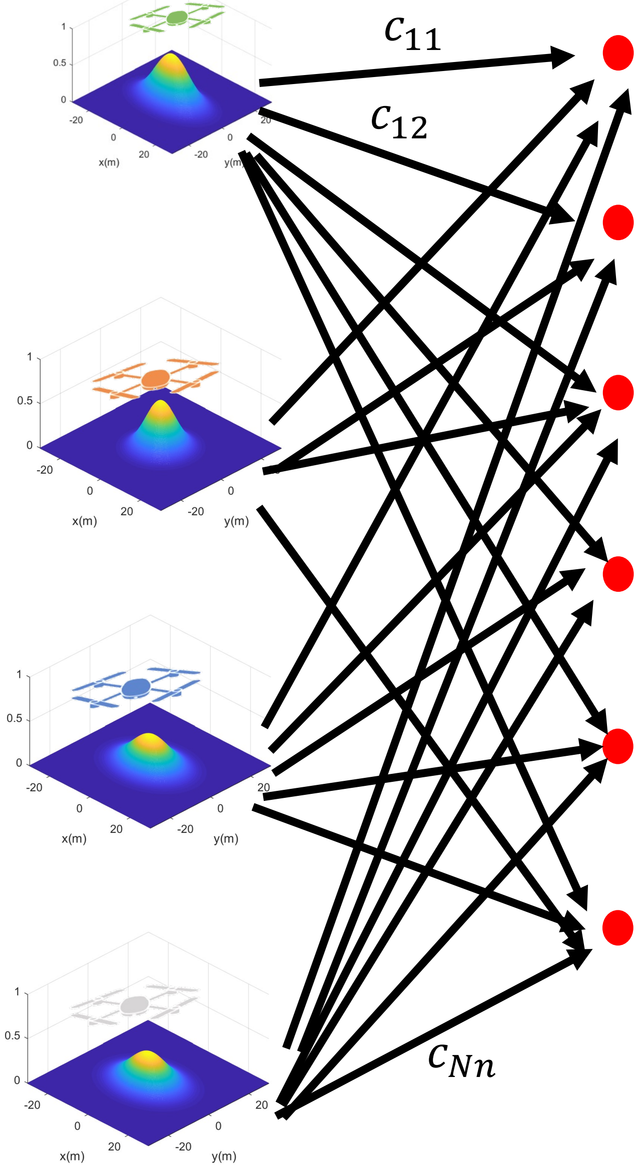
Stien Coverage

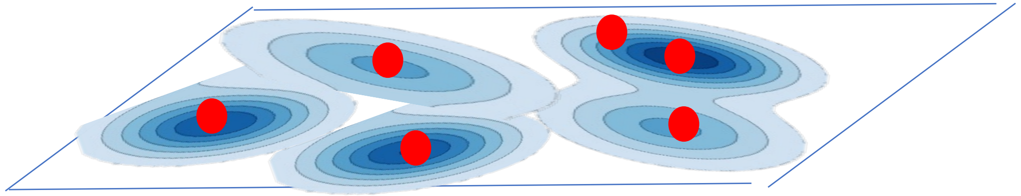
- ➢ Identifies important points to start the sweep from
- ➢ The points can be given to Voronoi partitioning to divide the area.

Summary

$\mathcal{A}$:Agent set

$\mathcal{S}$: set of points of interest (PoIs)





Use SVGD to find PoIs

Optimal Agent Assignment

$$\mathbf{Z}^\star = \arg\min \sum_{j\in\mathcal{S}} \sum_{i\in\mathcal{A}} Z_{i,j} C^\star_{i,j},$$

$$Z_{i,j} \in \{0,1\}, \quad i \in \mathcal{A},\ j \in \mathcal{S},$$

$$\sum_{i\in\mathcal{A}} Z_{i,j} = 1,\ \forall i \in \mathcal{A},$$

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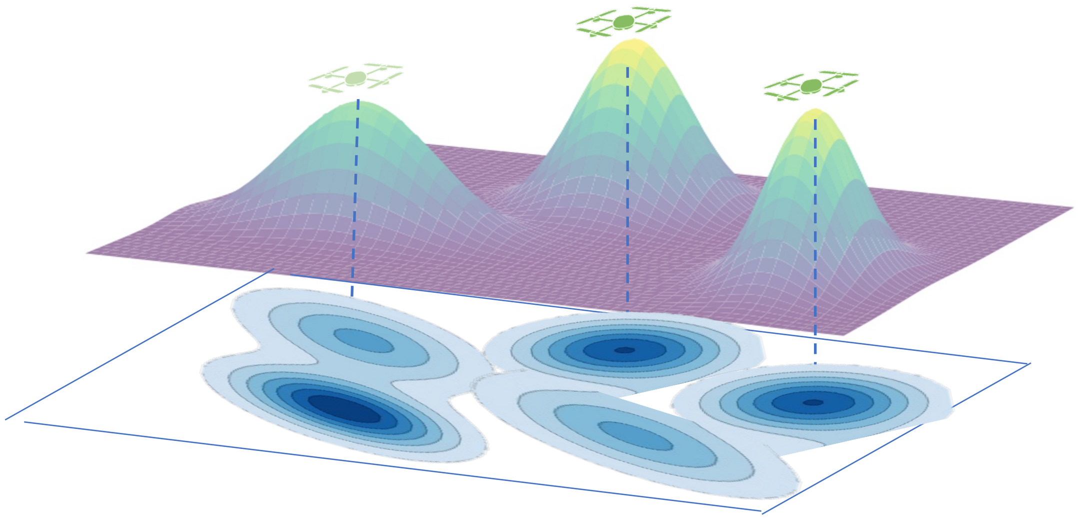

$$C^\star_{i,j} = \min_{\theta_i\in\Theta} \left\{ \mathcal{KL}\left(s(x|x_j,\theta_i) || p(x) \right)\ for\ x \in \mathcal{C}_i(x|x_j,\theta_i) \right\}$$

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Student Contributors

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