A Centralized-equivalent Decentralized Implementation of Extended Kalman Filters for Cooperative Localization

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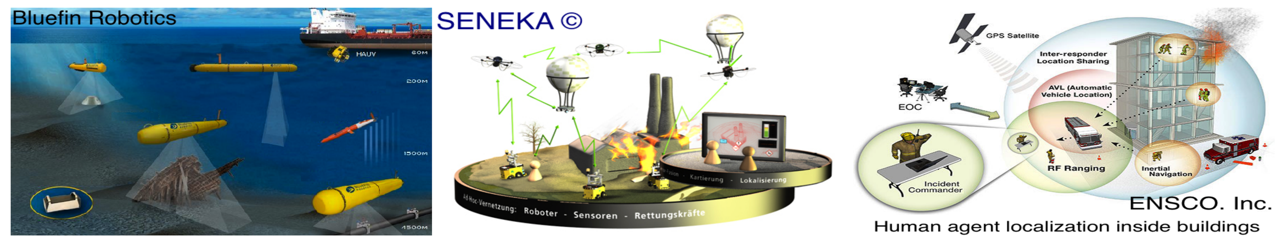
Problem: localize a team of mobile agents

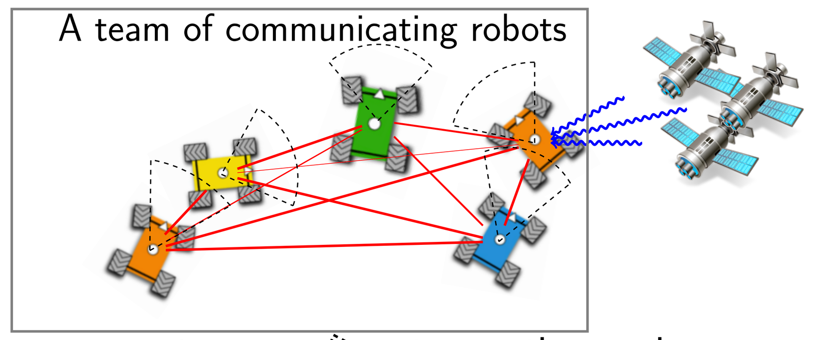


- Environment is not accessible a priori: cannot use beacon-based localization
- Environment is changing: cannot use SLAM
- Environment is fully or partially GPS denied

How to localize?

Cooperative localization (CL)

How to localize? Use cooperative localization



Use relative measurements among the mobile agents as a feedback signal to jointly estimate the poses of the team members

Decentralized cooperative localization (D-CL)

- Centralized CL can be easier to design **BUT**
  - Single failure point
  - High communication and computation cost
  - Not scalable
- Preferred operation: decentralized operation

Major challenge to develop decentralized CL: how to keep an accurate account of cross-correlations without all-to-all communication

Robotic team description

A group of *heterogeneous* mobile robots with computation, communication and measurement capabilities

- robot's equations of motion

$$x(k+1)=\begin{bmatrix}x^1(k+1)\\ \vdots\\ x^N(k+1)\end{bmatrix}=\begin{bmatrix}f^1(x^1(k),u^1(k))+g(x^1(k))n^1(k)\\ \vdots\\ f^N(x^N(k),u^N(k))+g(x^N(k))n^N(k)\end{bmatrix},$$

process noise: $E[n^i]=0,\ E[n^in^i]=Q^i,\ E[n^in^j]=0$

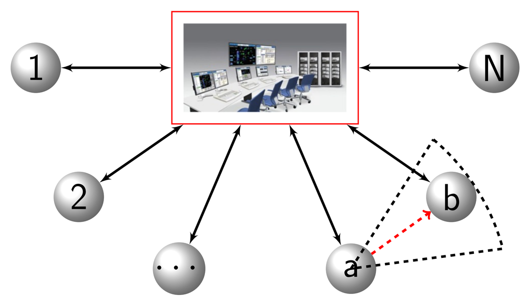
- measurement model

relative measurement: $z_{ij}(k)=h_{ij}(x^i(k),x^j(k))+\nu^i(k)$

measurement noise: $E[\nu^i]=0,\ E[\nu^i\nu^i]=R^i,\ E[\nu^i\nu^j]=0$

- bounded communication range is bigger than bounded measurement range

Centralized extended Kalman filter for cooperative localization



a takes measurement form b

Propagation

- For all $i\in\{1,\dots,N\}$

$$\begin{aligned}\hat{x}^{i-}(k+1)&=f^i(\hat{x}^{i+}(k),u^i(k))\\ P^{i-}(k+1)&=F^i(k)P^{i+}(k)F^i(k)^\top\\ &\quad+G^i(k)Q^i(k)G^i(k)^\top\\ P^-_{ij}(k+1)&=F^i(k)P^+_{ij}(k)F^j(k)^\top\\ &\qquad j\in\{1,\dots,N\}\backslash\{i\}\end{aligned}$$

Update due to relative measurement

$$\begin{aligned}r^a&=z_{ab}-h_{ab}(\hat{x}^{a-}(k+1),\hat{x}^{b-}(k+1))\\ S_{ab}&=R^a(k+1)+\tilde{H}_a(k+1)P^{a-}(k+1)\tilde{H}_a(k+1)^\top\\ &\quad+\tilde{H}_b(k+1)P^{b-}(k+1)\tilde{H}_b(k+1)^\top\\ &\quad-\tilde{H}_b(k+1)P^-_{ba}(k+1)\tilde{H}_a(k+1)^\top\\ &\quad-\tilde{H}_a(k+1)P^-_{ab}(k+1)\tilde{H}_b(k+1)^\top\end{aligned}$$

- update gain K_i of robot $i\in\{1,\dots,N\}$

$$K_i=(P^-_{ib}(k+1)\hat{H}_b^\top-P^-_{ia}(k+1)\hat{H}_a^\top)S_{ab}{}^{-1}$$

- For all $i\in\{1,\dots,N\}$

$$\begin{aligned}\hat{x}^{i+}(k+1)&=\hat{x}^{i-}(k+1)+K_ir^a\\ P^{i+}(k+1)&=P^{i-}(k+1)-K_iS_{ab}K_i^\top\\ P^+_{ij}(k+1)&=P^-_{ij}(k+1)-K_iS_{ab}K_j^\top\\ &\qquad j\in\{1,\dots,N\}\backslash\{i\}\end{aligned}$$

Source of coupling: cross-covariance terms

Main result: *Interim Master D-CL*

Initialization

Every robot $i\in\{1,\dots,N\}$ at $k=0$

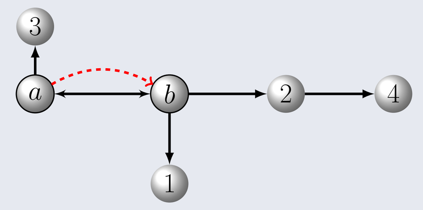
$$\begin{aligned}&\hat{x}^{i+}(0)\in\mathbb{R}^{n^i},\quad \Phi^i(0)=I_{n^i}\\ &P^{i+}(0)\in\mathbb{R}^{n^i\times n^i},\quad P^{i+}(0)>0\\ &\bar{P}^i_{lj}(0)=0,\ j\in\{1,\dots,N-1\},l\in\{j+1,\cdots,N\}\end{aligned}$$

Propagation

Every robot $i\in\{1,\dots,N\}$ propagates

$$\begin{aligned}\hat{x}^{i-}(k+1)&=f^i(\hat{x}^{i+},u^i(k))\\ P^{i-}(k+1)&=F^i(k)P^i(k^+)F^i(k)^\top+Q^i(k)\\ \Phi^i(k+1)&=F^i(k)\Phi^i(k)\end{aligned}$$

Update due to relative measurement



- The interim master, robot a, acquires

landmark-message $=\left(\hat{x}^{b-},P^{b-},\Phi^b\text{ at }k+1\right)$

and calculates

$$\begin{aligned}S_{ab}&=R^a+\tilde{H}_aP^{a-}\tilde{H}_a^\top+\tilde{H}_b^\top P^{b-}\tilde{H}_b\\ \bar{r}^a&=(S_{ab})^{-\frac{1}{2}}r^a=(S_{ab})^{-\frac{1}{2}}(z_{ab}-h_{ab}(\hat{x}^{b-},\hat{x}^{a-}))\\ &\quad-\tilde{H}_a\Phi^a\bar{P}^a_{ab}\Phi^{b\top}\tilde{H}_b^\top-\tilde{H}_b\Phi^b\bar{P}^a_{ba}\Phi^{a\top}\tilde{H}_a^\top\\ \bar{D}_a&=(\bar{P}^a_{ab}\Phi^{b\top}\tilde{H}_b^\top-(\Phi^a)^{-1}P^{a-}\tilde{H}_a^\top)S_{ab}{}^{-1/2}\\ \bar{D}_b&=((\Phi^b)^{-1}P^{b-}\tilde{H}_b^\top-\bar{P}^a_{ba}\Phi^{a\top}\tilde{H}_a^\top)S_{ab}{}^{-1/2}\end{aligned}$$

and broadcasts

update-message $=\left(a,b,\bar{r}^a,\bar{D}_a,\bar{D}_b,\Phi^{b\top}\tilde{H}_b^\top S_{ab}{}^{-1/2},\Phi^{a\top}\tilde{H}_a^\top S_{ab}{}^{-1/2}\text{ at }k+1\right)$

- Every robot $i\in\{1,\dots,N\}$ calculates

$$\begin{aligned}\bar{D}_j&=\bar{P}^i_{jb}\Phi^{b\top}\tilde{H}_b^\top S_{ab}{}^{-\frac{1}{2}}-\bar{P}^i_{ja}\Phi^{a\top}\tilde{H}_a^\top S_{ab}{}^{-\frac{1}{2}}\\ &\qquad j\in\{1,\dots,N\}\backslash\{a,b\}\end{aligned}$$

and updates

$$\begin{aligned}\hat{x}^{i+}&=\hat{x}^{i-}+\Phi^i\bar{D}_i\bar{r}^a=\hat{x}^{i-}+K_ir^a\\ P^{i+}&=P^{i-}-\Phi^i\bar{D}_i\bar{D}_i^\top(\Phi^i)^\top=P^{i-}-K_iS_{ab}K_i^\top\\ \bar{P}^i_{lj}&=\bar{P}^i_{lj}-\bar{D}_l\bar{D}_j^\top\\ &\qquad j\in\{1,\dots,N-1\},\ l\in\{j+1,\cdots,N\}\end{aligned}$$

The strategy to decentralize

- eliminate direct calculation of cross-covariances
- create the required cross-covariance terms using
  - local new intermediate variables
  - message received from the robot making the relative measurement

Decomposition for decentralization

- define $\Phi^i(k+1)=F^i(k)\Phi^i(k)$, $\Phi^i(0)=I$, then
- $P_{ij}(k+1^-)=\Phi^i(k+1)\bar{P}_{ij}(k)\Phi^j(k+1)^\top$, $\bar{P}_{ij}(0)=0$
- then in any update stage $K_i=\Phi^i(k+1)\bar{D}_iS_{ab}^{-1/2}$

$$\bar{P}_{lj}(k+1)=\bar{P}_{lj}(k)-\bar{D}_l\bar{D}_j$$

For decentralized operation: every robot keeps a local copy of $\bar{P}_{lj}$

Interim Master D-CL in operation

Interim master: the robot making a relative measurement

- acquires information, *landmark-message*, from interim landmark
- calculates intermediate variables
- broadcasts *update-message* to the rest of robots
- using *update-message* and their local variables, each robots obtains updates that matches those of central CL

Multiple synchronized relative measurement update: use sequential updating

Complexity analysis per robot

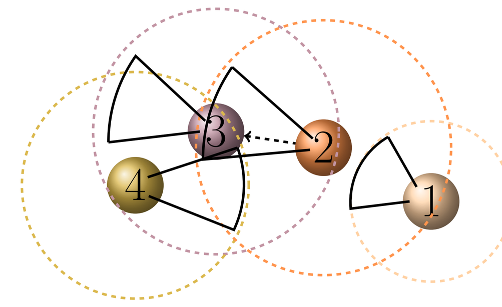
	Computation	Storage	Broadcast*	Message Size	Connectivity
Propagation	$O(1)$	$O(N^2)$	0	0	None
Update per N_z relative measur.	$O(N_z\times N^2)$	$O(N^2)$	$O(N_z)$	$O(1)$	spanning tree rooted at the master robots
Overall worst case ($N_z=N(N-1)$)	$O(N^4)$	$O(N^2)$	$O(N^2)$	$O(1)$	

*Broadcast cost is for multi-hop communication. If the communication range is unbounded, the broadcast cost per robot is at worst case of order $O(N)$.

Conclusion

Features of the algorithm

- a centralized-equivalent algorithm
- no communication at the propagation stage
- in update stage the communication graph only needs to have a spanning tree rooted at the master robot



- can easily incorporate updates due to absolute measurements
- robust to permanent agent drop out
- small communication message size, independent of the size of the network

Looking ahead

- extension of the algorithm to let new robots join the group
- study the effect of missed broadcast messages as well as asynchronous operation

Acknowledgment: this work was supported by

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