Singularly Perturbed Algorithms for Dynamic Average Consensus

Solmaz S. Kia, Jorge Cortés, Sonia Martínez

Mechanical and Aerospace Engineering Dept.
University of California San Diego
http://tintoretto.ucsd.edu/solmaz

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Dynamic Average Consensus

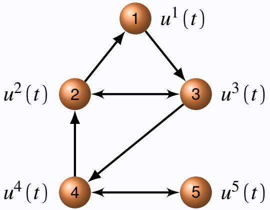
- Autonomous and cooperative agents

$$\dot{x}^i = -c^i, \quad x^i, c^i \in \mathbb{R}$$

- x^i: agreement state
- c^i: driving command

- Design $c^i = f(i, \text{neighbors})$ s.t. $\forall i \in \{1, \ldots, N\}$

$$x^i(t) \to \frac{1}{N} \sum_{j=1}^{N} u^j(t), \quad t \to \infty$$



Applications: distributed fusion of dynamic and evolving information

- multi-robot coordination
- distributed tracking
- sensor fusion
- feature-based map merging

Dynamic average consensus in the literature

Previous literature

- Focus on convergence to consensus
  - Specific initialization conditions: Spanos et al. 05, Zhu and Martinez 10
  - Specific set of inputs: Spanos et al. 05, Olfati-Saber and Shamma 05
  - Track with s.s. error: Olfati-Saber and Shamma 05, Spanos et al. 05, Freeman et al. 06, Zhu and Martinez 10
  - Require knowledge of the dynamics generating inputs: Bai et al. 10
  - Inputs with bounded derivatives: all of them
- No explicit attention to limited control authority
- No explicit attention to rate of convergence of individual agents

This talk

- Dynamic average consensus with pre-specified rate of convergence β:

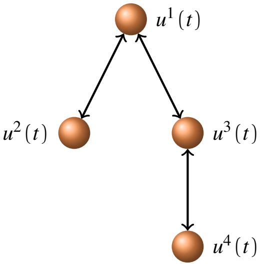
$$\left|x^i \to \frac{1}{N}\sum_{j=1}^{N} u^j(t)\right| \leqslant k\left|x^i(0) \to \frac{1}{N}\sum_{j=1}^{N} u^j(0)\right|\mathrm{e}^{-\beta t}$$

  - Network of agents with limited control authority; Control over time of arrival

- Dynamic average consensus with pre-specified rate of convergence β^i at each agent:

$$\left|x^i \to \frac{1}{N}\sum_{j=1}^{N} u^j(t)\right| \leqslant k\left|x^i(0) \to \frac{1}{N}\sum_{j=1}^{N} u^j(0)\right|\mathrm{e}^{-\beta^i t}$$

  - Network of agents with different levels of control authority; Control over time of arrival of each agent



Design tool: singular perturbation theory

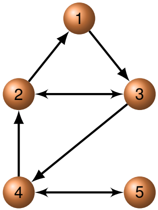
Communication topology: weighted digraph $\mathcal{G}(V, \mathcal{E}, A)$

- Node set: $V = \{1, \cdots, N\}$
- Edge set: $\mathcal{E} \subseteq V \times V$
- Weights (for $i, j \in \{1, \ldots, N\}$)

$$a_{ij} > 0 \textit{ if } (i,j) \in \mathcal{E}, \; a_{ij} = 0 \textit{ if } (i,j) \notin \mathcal{E}$$

- Strongly connected: $i \to j$ for any i, j
- Weight-balanced:

$$\sum_{j=1}^{N} a_{ji} = \sum_{j=1}^{N} a_{ij}, \quad i \in V$$



Goal: Dynamic average consensus with pre-specified rate of convergence β:

$$\left|x^i \to \frac{1}{N}\sum_{j=1}^{N} u^j(t)\right| \leqslant k\left|x^i(0) \to \frac{1}{N}\sum_{j=1}^{N} u^j(0)\right|\mathrm{e}^{-\beta t}$$

Design methodology

- Simplest dynamics: $x^i \to \frac{1}{N}\sum_{j=1}^{N} u^j(t)$ **with rate** β

$$\dot{x}^i = -\beta\left(x^i - \frac{1}{N}\sum_{j=1}^{N} u^j\right) + \frac{1}{N}\sum_{j=1}^{N} \dot{u}^j$$

- Requirement:
  - **fast** dynamics to generate $\beta\frac{1}{N}\sum_{j=1}^{N} u^j + \frac{1}{N}\sum_{j=1}^{N} \dot{u}^j$ in a **distributed manner** !

Dynamic average consensus with pre-specified rate of convergence

Goal: Dynamic average consensus with pre-specified rate of convergence β:

$$\left| x^i \rightarrow \frac{1}{N}\sum_{j=1}^{N} u^j(t) \right| \leqslant k \left| x^i(0) \rightarrow \frac{1}{N}\sum_{j=1}^{N} u^j(0) \right| \mathrm{e}^{-\beta t}$$

Design methodology

- Simplest dynamics: $x^i \rightarrow \frac{1}{N}\sum_{j=1}^{N} u^j(t)$ **with rate** β

$$\dot{x}^i = -\beta\left(x^i - \frac{1}{N}\sum_{j=1}^{N} u^j\right) + \frac{1}{N}\sum_{j=1}^{N} \dot{u}^j$$

- Requirement:
  - **fast** dynamics to generate $\beta\,\frac{1}{N}\sum_{j=1}^{N} u^j + \frac{1}{N}\sum_{j=1}^{N} \dot{u}^j$ in a **distributed manner** !

Design methodology

- Desired dynamics: $\dot{x}^i = -\big(\beta x^i - (\beta \frac{1}{N}\sum_{j=1}^{N} u^j + \frac{1}{N}\sum_{j=1}^{N} \dot{u}^j)\big)$
- Requirement: **fast** distributed dynamics to generate $\beta \frac{1}{N}\sum_{j=1}^{N} u^j + \frac{1}{N}\sum_{j=1}^{N} \dot{u}^j$
- Two-time scale algorithm:
  - Initialize at $k=0$, $x^i(0) \in \mathbb{R}$
  - **Fast dynamics**: at each time k, obtain $u^i(k)$ and $\dot{u}^i(k)$. $\forall i \in \{1,\dots,N\}$, run

$$\begin{cases} \dot{z}^i = -(z^i - \beta u^i(k) - \dot{u}^i(k)) - \sum_{i=1}^{N} a_{ij}(z^i - z^j) - \sum_{i=1}^{N} a_{ij}(v^i - v^j) \\ \dot{v}^i = \sum_{i=1}^{N} a_{ji}(z^i - z^j) \end{cases}$$

$$z^i(t,k) \to \beta \frac{1}{N}\sum_{j=1}^{N} u^j(k) + \frac{1}{N}\sum_{j=1}^{N} \dot{u}^j(k), \quad \text{exponentially as } t \to \infty, \text{ (Due to [a])}$$

  - **Slow dynamics**: $x^i(k+1) = x^i(k) - \Delta t(\beta x^i(k) - z^i(k))$
  - $k \leftarrow k+1$

[a] R. Freeman et al., "Stability and convergence properties of dynamic average consensus estimators," CDC 2006

Dynamic average consensus with pre-specified rate of convergence

Design methodology

- Desired dynamics: $\dot{x}^i = -\big(\beta x^i - (\beta \frac{1}{N} \sum_{j=1}^N u^j + \frac{1}{N} \sum_{j=1}^N \dot{u}^j)\big)$
- Requirement: **fast** distributed dynamics to generate $\beta \frac{1}{N} \sum_{j=1}^N u^j + \frac{1}{N} \sum_{j=1}^N \dot{u}^j$
- Two-time scale algorithm:
  - Initialize at $k = 0$, $x^i(0) \in \mathbb{R}$
  - **Fast dynamics**: at each time k, obtain $u^i(k)$ and $\dot{u}^i(k)$. $\forall i \in \{1, \dots, N\}$, run

$$\begin{cases} \dot{z}^i = -(z^i - \beta u^i(k) - \dot{u}^i(k)) - \sum_{i=1}^N a_{ij}(z^i - z^j) - \sum_{i=1}^N a_{ij}(\nu^i - \nu^j) \\ \dot{\nu}^i = \sum_{i=1}^N a_{ji}(z^i - z^j) \end{cases}$$

$$z^i(t,k) \to \beta \frac{1}{N} \sum_{j=1}^N u^j(k) + \frac{1}{N} \sum_{j=1}^N \dot{u}^j(k), \quad \text{exponentially as } t \to \infty, \text{ (Due to [a])}$$

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An average consensus with pre-specified rate of coverage

- **Fast dynamics**: at each time k, $\forall i \in \{1, \ldots, N\}$

$$\begin{cases} \dot{z}^i = -(z^i - \beta u^i(k) - \dot{u}^i(k)) - \sum_{i=1}^{N} a_{ij}(z^i - z^j) - \sum_{i=1}^{N} a_{ij}(\nu^i - \nu^j) \\ \dot{\nu}^i = \sum_{i=1}^{N} a_{ji}(z^i - z^j) \end{cases}$$

- **Slow dynamics**: $x^i(k+1) = x^i(k) - \Delta t(\beta x^i(k) - z^i(k))$

Innovation

- Combine the fast and slow dynamics in one continuous-time algorithm
- No need to wait for fast dynamics to converge to take steps in the slow dynamics
- Design is based on singularly perturbed systems

Singular perturbation

$$\begin{cases} \dot{x} = f(t,x,z), & x \in \mathbb{R}^n \\ \epsilon \dot{z} = g(t,x,z), & z \in \mathbb{R}^m, \end{cases} \xrightarrow{\epsilon = 0} \begin{cases} \dot{x} = f(t,x,z), & x \in \mathbb{R}^n \\ 0 = g(t,x,z) \end{cases}$$

Slow dynamics:

$$g(t,x,z) = 0 \Rightarrow z = h(t,x)$$

$$\dot{x} = f(t,x,h(t,x))$$

Fast dynamics: fixed $(t, x(t))$ and $\tau = t/\epsilon$

$$\frac{dz}{d\tau} = g(t,x,z)$$

Singular perturbation

$$\begin{cases} \dot{x} = f(t, x, z), & x \in \mathbb{R}^n \\ \epsilon \dot{z} = g(t, x, z), & z \in \mathbb{R}^m, \end{cases} \quad \xrightarrow{\epsilon = 0} \quad \begin{cases} \dot{x} = f(t, x, z), & x \in \mathbb{R}^n \\ 0 = g(t, x, z) \end{cases}$$

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Slow dynamics:

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Singular perturbation

$$\begin{cases} \dot{x} = f(t,x,z), & x \in \mathbb{R}^n \\ \epsilon \dot{z} = g(t,x,z), & z \in \mathbb{R}^m, \end{cases} \quad \xrightarrow{\epsilon = 0} \quad \begin{cases} \dot{x} = f(t,x,z), & x \in \mathbb{R}^n \\ 0 = g(t,x,z) \end{cases}$$

Slow dynamics:

$$g(t,x,z) = 0 \Rightarrow z = h(t,x)$$

$$\dot{x} = f(t,x,h(t,x))$$

Fast dynamics: fixed $(t, x(t))$ and $\tau = t/\epsilon$

$$\frac{dz}{d\tau} = g(t,x,z)$$

Singular perturbation on infinite intervals

Full dynamics:

$$\begin{cases} \dot{x} = f(t,x,z), \\ \epsilon\dot{z} = g(t,x,z) \end{cases}$$

$$\text{Solution}: x(t,\epsilon)$$

Slow and fast dynamics

$$\begin{cases} \dot{x} = f(t,x,h(t,x)) \\ \frac{dz}{d\tau} = g(t,x,z) \end{cases}$$

$$\text{Solution}: \bar{x}(t)$$

Theorem

For $[t, x, z-h(t,x), \epsilon] \in [0,\infty) \times D_x \times D_y \times [0,\epsilon_0)$

- *On any compact subset of* $D_x \times D_y$*:*
  - *continuous and bounded:* $f,\ g,\ \partial f|_{\partial x, \partial z, \partial\epsilon},\ \partial g|_{\partial x,\partial z,\partial\epsilon,\partial t}$
  - *bounded partial derivative w.r.t arg:* $h(t,x),\ \partial g(t,x,z,0)/\partial z$
  - $\partial f(t,x,h(t,x),0)/\partial x$ *is Lipschitz in* x*, uniformly in* t*,*
- *The slow dynamics is exponentially stable*
- *The fast dynamics is exponentially stable*

For any $t_0 \geqslant 0$*,* $\exists\, \epsilon^\star$ *s.t. for* $0 < \epsilon \leqslant \epsilon^\star$ *we have*

$$x(t,\epsilon) - \bar{x}(t) \in O(\epsilon), \quad t \in [t_0, \infty)$$

Proposed dynamic average consensus algorithms with pre-specified rate of convergence

A singularly perturbed dynamic average consensus: pre-specified rate of coverage β

- 1st-Order-Input Dynamic Consensus (FOI-DC)

$$\begin{cases} \epsilon\, \dot{z}^i = -(z^i + \beta\, u^i + \dot{u}^i) - \sum_{i=1}^{N} a_{ij}(z^i - z^j) - \sum_{i=1}^{N} a_{ij}(\nu^i - \nu^j), \\ \epsilon\, \dot{\nu}^i = \sum_{i=1}^{N} a_{ji}(z^i - z^j), \end{cases}$$
$$\dot{x}^i = -\beta\, x^i - z^i,$$

- 2nd-Order-Input Dynamic Consensus (SOI-DC)

$$\begin{cases} \epsilon\, \dot{z}^i = -(z^i + \beta\, u^i + \dot{u}^i) - \sum_{i=1}^{N} a_{ij}(z^i - z^j) - \sum_{i=1}^{N} a_{ij}(\nu^i - \nu^j) - \epsilon(\beta\, \dot{u}^i + \ddot{u}^i), \\ \epsilon\, \dot{\nu}^i = \sum_{i=1}^{N} a_{ji}(z^i - z^j), \end{cases}$$
$$\dot{x}^i = -\beta\, x^i - z^i,$$

Convergence analysis: using singular perturbation theory

Convergence analysis of the proposed algorithms

- FOI-DC

$$\begin{cases} \epsilon\, \dot{z}^i = -(z^i + \beta\, u^i + \dot{u}^i) - \sum_{i=1}^N a_{ij}(z^i - z^j) - \sum_{i=1}^N a_{ij}(\nu^i - \nu^j), \\ \epsilon\, \dot{\nu}^i = \sum_{i=1}^N a_{ji}(z^i - z^j), \end{cases}$$
$$\dot{x}^i = -\beta\, x^i - z^i,$$

- SOI-DC

$$\begin{cases} \epsilon\, \dot{z}^i = -(z^i + \beta\, u^i + \dot{u}^i) - \sum_{i=1}^N a_{ij}(z^i - z^j) - \sum_{i=1}^N a_{ij}(\nu^i - \nu^j) - \epsilon(\beta\, \dot{u}^i + \ddot{u}^i), \\ \epsilon\, \dot{\nu}^i = \sum_{i=1}^N a_{ji}(z^i - z^j), \end{cases}$$
$$\dot{x}^i = -\beta\, x^i - z^i,$$

Theorem

- *Let $\mathcal{G}$ be a strongly connected and weight-balanced digraph*
- *FOI-DC: $\dot{u}^i$ and $\ddot{u}^i$ continuous and bounded for $t \geqslant 0$*
- *SOI-DC: $\dot{u}^i$ and $\dddot{u}^i$ are continuous and bounded for $t \geqslant 0$*

Then, $\exists \epsilon^\star > 0$ s. t., for all $\epsilon \in (0, \epsilon^\star]$, $x^i(0), z^i(0), \nu^i(0) \in \mathbb{R}$, $\forall i \in \{1, \ldots, N\}$

$$\left|x^i(t,\epsilon) - \frac{1}{N}\sum_{j=1}^N u^j(t)\right| < O(\epsilon) + \left|x^i(0) - \frac{1}{N}\sum_{j=1}^N u^j(0)\right| e^{-\beta t},$$

Sketch of proof:

- Fast dynamics is exponentially stable ($\tau = t/\epsilon$)

$$\begin{cases} dz^i/d\tau = -(z^i + \beta\, u^i + \dot{u}^i) - \sum_{i=1}^{N} a_{ij}(z^i - z^j) - \sum_{i=1}^{N} a_{ij}(\nu^i - \nu^j), \\ d\nu^i/d\tau = \sum_{i=1}^{N} a_{ji}(z^i - z^j), \end{cases}$$

Setting $\epsilon = 0$: $z^i = \beta \frac{1}{N} \sum_{j=1}^{N} u^j + \frac{1}{N} \sum_{j=1}^{N} \dot{u}^j$, $\forall i \in \{1, \ldots, N\}$

- Slow dynamics is exponentially stable

$$\dot{x}^i = -\beta \left(x^i - \frac{1}{N} \sum_{j=1}^{N} u^j \right) + \frac{1}{N} \sum_{j=1}^{N} \dot{u}^j \rightarrow \left| x^i(t) - \frac{1}{N} \sum_{j=1}^{N} u^j(t) \right| \leqslant \left| x^i(0) - \frac{1}{N} \sum_{j=1}^{N} u^j(0) \right| \mathrm{e}^{-\beta t}$$

- Lipschitz and continuity conditions are satisfied

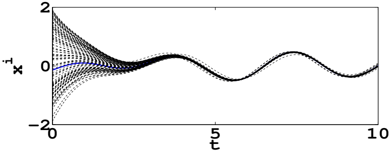
$$\therefore \quad \boldsymbol{x}(t, \epsilon) - \boldsymbol{x}(t) \in O(\epsilon)$$

- $N = 100$
- random connected graph
- $u^i(t) = a^i \, \sin(b^i \, t + c^i)$

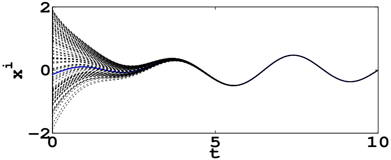
  $a^i \sim \mathcal{U}[-5, 5]$

  $b^i \sim \mathcal{U}[1, 2]$

  $c^i \sim \mathcal{U}[0, \pi/2]$



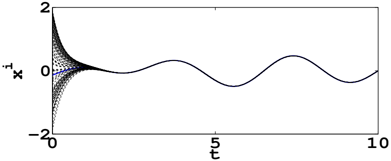
$\epsilon = 0.01$ and $\beta = 1$



$\epsilon = 0.001$ and $\beta = 1$

$\downarrow \epsilon$: $\downarrow$ **error**

$\uparrow \beta$: **faster convergence**



$\epsilon = 0.001$ and $\beta = 3$

Convergence analysis of SOI-DC for inputs differing by static values

Lemma

$$\begin{cases} \epsilon\,\dot{z}^i = -(z^i + \beta\,u^i + \dot{u}^i) - \sum_{i=1}^{N} a_{ij}(z^i - z^j) - \sum_{i=1}^{N} a_{ij}(\nu^i - \nu^j) - \epsilon(\beta\,\dot{u}^i + \ddot{u}^i), \\ \epsilon\,\dot{\nu}^i = \sum_{i=1}^{N} a_{ji}(z^i - z^j), \end{cases}$$

$$\dot{x}^i = -\beta\,x^i - z^i,$$

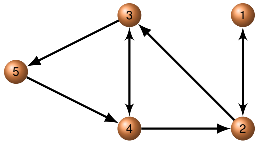
- *Let $\mathcal{G}$ be a strongly connected and weight-balanced digraph*
- $u^i(t) = u(t) + \bar{u}^i$, $\dot{\bar{u}}^i = 0$, $\forall i \in \{1, \ldots, N\}$

Then, $\forall \epsilon > 0$ *and* $\beta > 0$ *and,* $x^i(0), z^i(0), \nu^i(0) \in \mathbb{R}$, $\forall i \in \{1, \ldots, N\}$

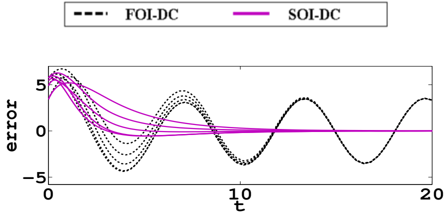
$$x^i(t, \epsilon) \to \frac{1}{N}\sum_{j=1}^{N} u^j(t), \quad t \to \infty$$

Proof is based on Lyapunov approach!

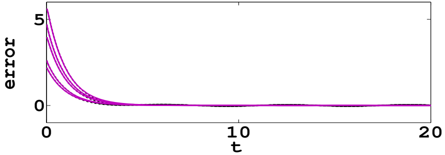
Simulation: SOI-DC for inputs differing by static values



$$u^1(t) = 5\sin(t) + 1,$$
$$u^2(t) = 5\sin(t) - 1,$$
$$u^3(t) = 5\sin(t) + 4,$$
$$u^4(t) = 5\sin(t) + 5,$$
$$u^5(t) = 5\sin(t) + 10.$$



$\epsilon = 1$ and $\beta = 1$



$\epsilon = 0.01$ and $\beta = 1$

SOI-DC tracks regardless of value of ϵ

A dynamic average consensus with pre-specified rate of convergence at each agent

Lemma

$$\begin{cases} \epsilon\,\dot{z}^i = -(z^i + u^i) - \sum_{i=1}^{N} a_{ij}(z^i - z^j) - \sum_{i=1}^{N} a_{ij}(\nu^i - \nu^j), \\ \epsilon\,\dot{\nu}^i = \sum_{i=1}^{N} a_{ji}(z^i - z^j), \end{cases}$$

$$\begin{cases} \epsilon\,\dot{y}^i = -(y^i + \dot{u}^i) - \sum_{i=1}^{N} a_{ij}(y^i - y^j) - \sum_{i=1}^{N} a_{ij}(\mu^i - \mu^j), \\ \epsilon\,\dot{\mu}^i = \sum_{i=1}^{N} a_{ji}(y^i - y^j), \end{cases}$$

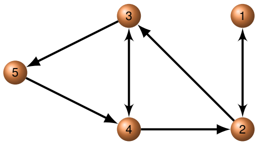
$$\dot{x}^i = -\beta^i x^i - \beta^i z^i - y^i,$$

- *Let $\mathcal{G}$ be a strongly connected and weight-balanced digraph*
- *Assume $\dot{u}^i$ and $\ddot{u}^i$ continuous and bounded for $t \geqslant 0$*

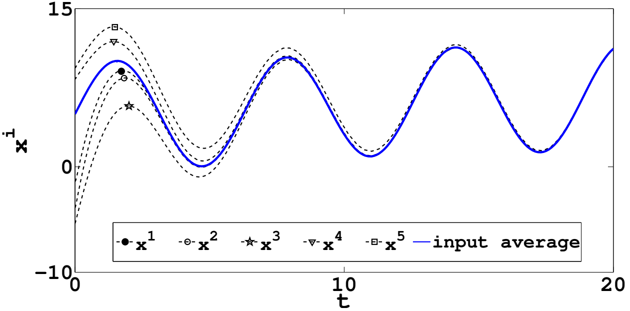
Then, $\forall \epsilon > 0$ *and* $\beta^i > 0$ *and,* $x^i(0), z^i(0), \nu^i(0) \in \mathbb{R}$, $\forall i \in \{1, \dots, N\}$

$$\left|x^i(t,\epsilon) - \frac{1}{N}\sum_{j=1}^{N} u^j(t)\right| < O(\epsilon) + \left|x^i(0) - \frac{1}{N}\sum_{j=1}^{N} u^j(0)\right| e^{-\beta^i t},$$

Simulation: agents set their own rate of convergence



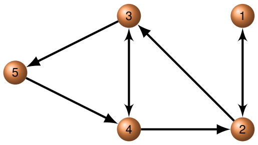
$u^1(t) = 5\sin(t) + \frac{10}{t+2} + 1,$
$u^2(t) = 5\sin(t) + 0.5t - 1,$
$u^3(t) = 5\sin(t) + \cos(0.5t) + 4,$
$u^4(t) = 5\sin(t) + \log(t+1) + 5,$
$u^5(t) = 5\sin(t) + \text{atan}(t) + 10.$



$\epsilon = 0.01$ and $\beta^1 = 1.2,\ \beta^2 = 1,\ \beta^3 = 0.5,\ \beta^4 = 0.4,\ \beta^5 = 0.2$

Control over rate of convergence ~ Control over time of arrival!

Simulation: agents set their own rate of convergence



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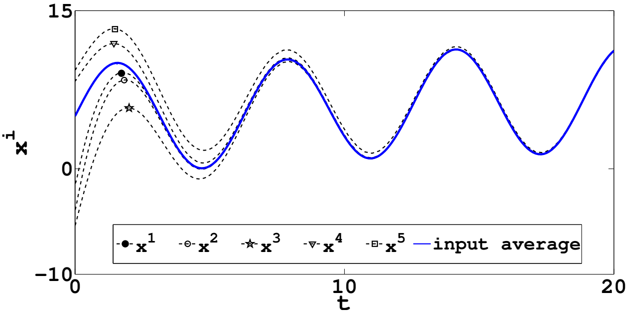
$$u^2(t) = 5\sin(t) + 0.5t - 1,$$
$$u^3(t) = 5\sin(t) + \cos(0.5t) + 4,$$
$$u^4(t) = 5\sin(t) + \log(t+1) + 5,$$
$$u^5(t) = 5\sin(t) + \text{atan}(t) + 10.$$



$\epsilon = 0.01$ and $\beta^1 = 1.2,\ \beta^2 = 1,\ \beta^3 = 0.5,\ \beta^4 = 0.4,\ \beta^5 = 0.2$

Control over rate of convergence ~ Control over time of arrival!

Conclusion

Summary

- We presented a distributed dynamic average consensus algorithm with pre-specified rate of convergence
- We provided a variation which allows each agent to choose its own rate of convergence
- Our algorithm is suitable for networked systems with limited control authority

Future work

- Quantifying the $O(\epsilon)$
- Rigorous treatment of switching topologies
- Relaxing boundedness and continuity conditions of $(\dot{u}^i, \ddot{u}^i)$

tintoretto.ucsd.edu/solmaz/

skia@ucsd.edu

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