A sub-modular receding horizon approach to persistent monitoring for a group of mobile agents over an urban area

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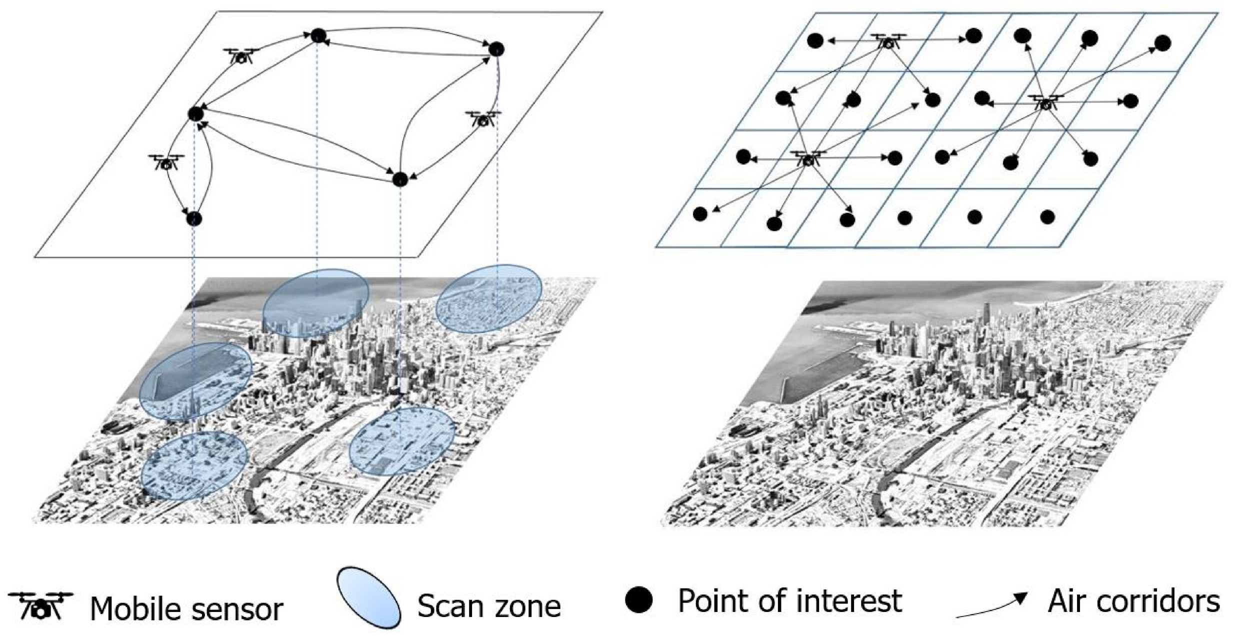
Abstract

The problem of interest: persistent monitoring of finite number of inter-connected geographical nodes for event detection via a group of heterogeneous mobile agents.

1- Poisson process model is used to capture the probability of the events occurring at the geographical nodes.
2- Reward function is tied to expected value of number of events discovered during the mission.
3- Finding the long term optimal solution is computationally infeasible (NP-hard problem). A receding horizon approach is proposed to overcome the issue.
4- Nodal importance is introduced to overcome shortsightedness of receding horizon approach.
5- Submodularity of the reward function leads to a sub-optimal distributed solution with guaranteed optimality gap.

Persistent Monitoring (*PM*)

A team of communicating mobile agents



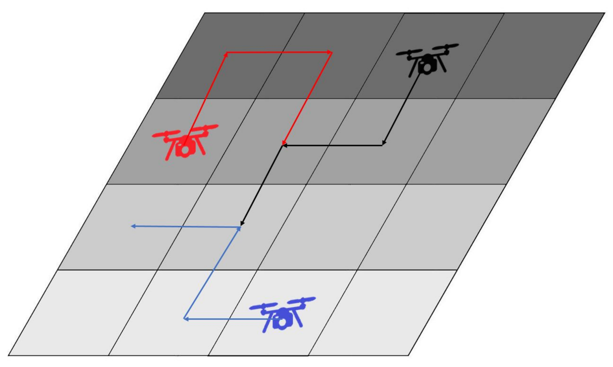
Objective: Use the capacity of the agents to find the event of interest in a map.

- Complexity increases with the monitoring time span

also

- One shot optimization does not have the robustness with respect to mission uncertainities.

Problem settings



Poisson based grid reward

$$R_i(t) = \begin{cases} 0, & t = t_{s,i}, \\ \psi_i(t - t_{s,i}), & t > t_{s,i}, \end{cases}$$

Sequence of nodes to visit: $n^j = \left[\mathsf{n}^j(0), \mathsf{n}^j(1), \cdots, \mathsf{n}^j(N_f^j)\right]^\top$

Time to visit nodes: $T^j = \left[T^j(0), T^j(1), \cdots, T^j(N_f^j)\right]$

Agent j: j

Sub-Policy of agent j: $\mathsf{p} = (n^j, T^j, j), \quad \mathcal{P}^j = \cup \mathsf{p}$

Total reward collected: $\mathsf{R}(\bar{\mathcal{P}}) = \sum_{\forall \mathsf{p} \in \bar{\mathcal{P}}} \sum_{j=1}^{|n_\mathsf{p}|} R_{n_\mathsf{p}(j)}(T_\mathsf{p}(j))$

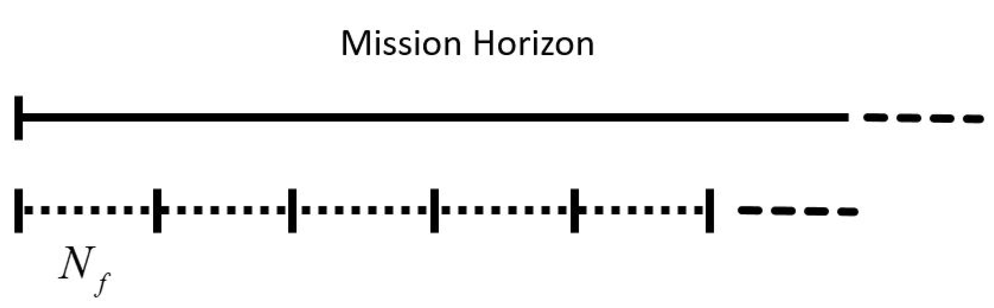
Question: How to choose $\bar{\mathcal{P}} \subset \mathcal{P} = \cup_{j=1}^{M} \mathcal{P}^j$ such that R is maximized?

Note: At most one sub-policy from policy set of each agent is chosen: $|\bar{\mathcal{P}} \cap \mathcal{P}^i| \leq 1$

$$\mathcal{P}^\star = \underset{\bar{\mathcal{P}} \subset \mathcal{P}}{\mathrm{argmax}}\, \mathsf{R}(\bar{\mathcal{P}}), \quad \text{s.t.} \quad |\bar{\mathcal{P}} \cap \mathcal{P}^i| \leq 1,$$

Receding Horizon Approach

Objective: Maximizing the mobiles agent's utility



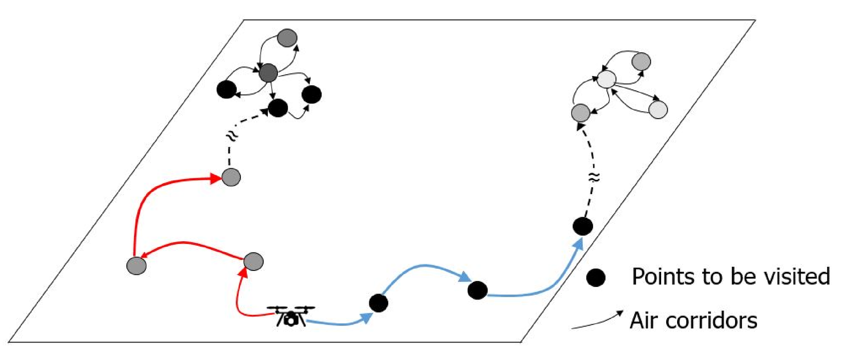
Note: As the mission horizon increases finding the optimal patrolling scheme becomes computationally intractable.

Approach: Breaking the mission horizon into smaller planning horizons

- Not globally optimal

but

- Robust to operational uncertainties



- Receding horizon approach makes the algorithm short sighted to the distant areas of the map.
- Nodal importance term is added to R is compensate for the shortsightedness.

$$\bar{\mathsf{R}}(\mathcal{P}) = \mathsf{R}(\mathcal{P}) + \alpha \sum_{\forall \mathsf{p} \in \bar{\mathcal{P}}} \mathsf{L}^\star(\mathsf{p}), \qquad \alpha \in \mathbb{R}_{\geq 0},$$

Submodular Optimization

Theorem(Submodularity of the reward function $\bar{\mathsf{R}}$)

Let $\mathcal{P} = \bigcup_{i=1}^{M} \mathcal{P}^i$ be the set of the union of the feasible policies of the agents, $\mathcal{P}^i$, $i \in \{1, \cdots, M\}$, over the horizon N_H. Then, for any $\alpha \in \mathbb{R}_{\geq 0}$, the reward function $\bar{\mathsf{R}}$ is a monotone increasing and submodular set function over $\mathcal{P}$, i.e., $\bar{\mathsf{R}} : 2^{\mathcal{P}} \to$ is monotone increasing and submodular.

Sequential Greedy Algorithm

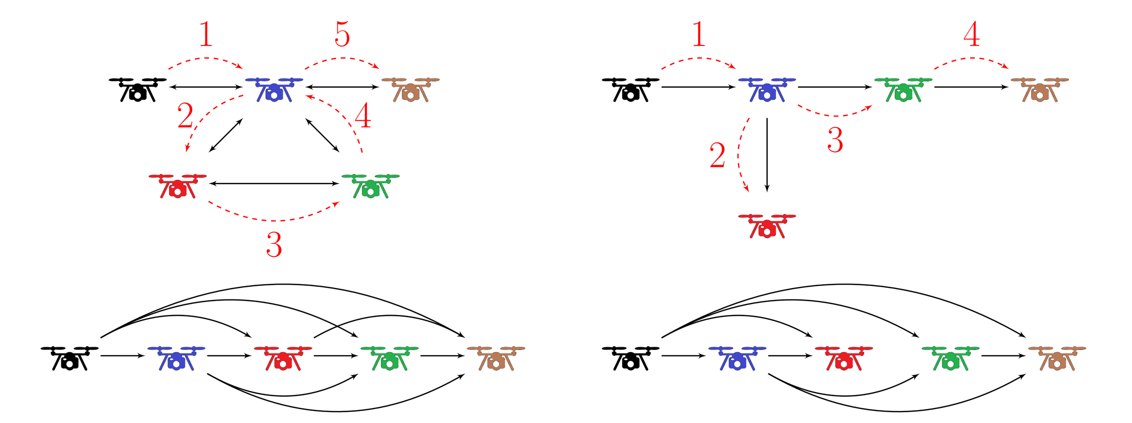
Theorem(Submodularity optimization

Let $\bar{\mathsf{R}}$ be a monotone submodular set function. Suppose $\bar{\mathsf{R}}(\mathcal{P}^\star)$ is the global maximum. Also, let $\mathcal{P}_{\mathsf{SG}}(M) \subset \mathcal{P}$ be the output of sequential greedy policy selection. Then, $g(\mathcal{P}_{\mathsf{SG}}(M)) \geq \frac{1}{2} g(\mathcal{P}^\star)$.

Algorithm 1 Sequential Greedy Algorithm

procedure SGOpt($\mathcal{P}^1, \cdots, \mathcal{P}^M, M$)
  Init: $\bar{\mathcal{P}} \leftarrow \emptyset,\ i \leftarrow 0$
  loop:
  while $i \leq M$ **do**
    $\mathsf{p}^{i\star} = \underset{\mathsf{p} \subset \mathcal{P}^i}{\mathrm{argmax}} \quad \Delta_{\bar{\mathsf{R}}}(\mathsf{p}|\bar{\mathcal{P}})$.
    $\bar{\mathcal{P}} \leftarrow \bar{\mathcal{P}} \cup \mathsf{p}^{i\star}$.
    $i \leftarrow i + 1$.
  end while
  Return $\bar{\mathcal{P}}$.
end procedure

Decentralized Solution

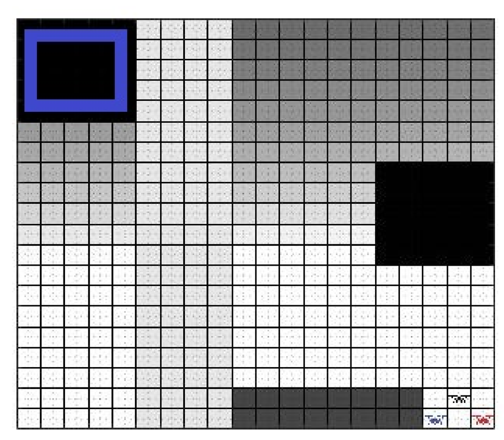


(a) $\mathcal{G}^a$ is a bidirectional connected graph. $\mathcal{G}^I$ is a complete graph.

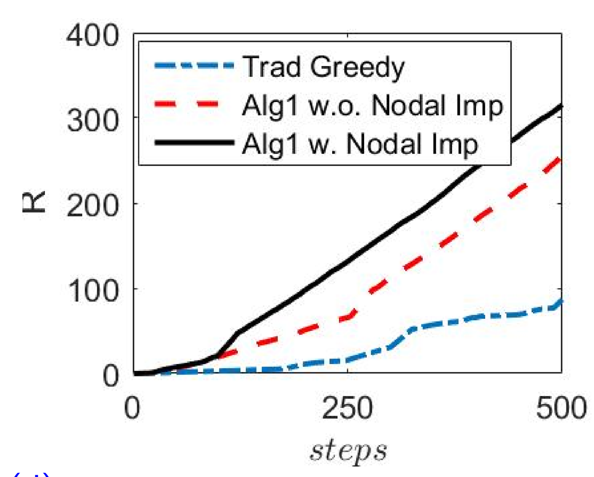
(b) $\mathcal{G}^a$ is a directed graph. $\mathcal{G}^I$ is an incomplete graph with clique number of 3.

- Submodular optimization has the inherent characteristic of decentralized implementation.
- Full information flow gives the optimality bound of $\frac{1}{2}$, otherwise the optimality bound is determined by the clique number of the graph.

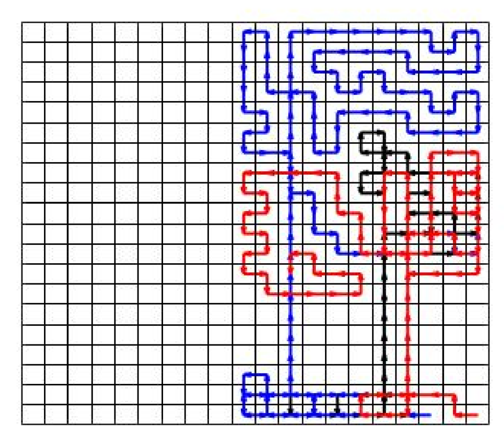
Numerical Results



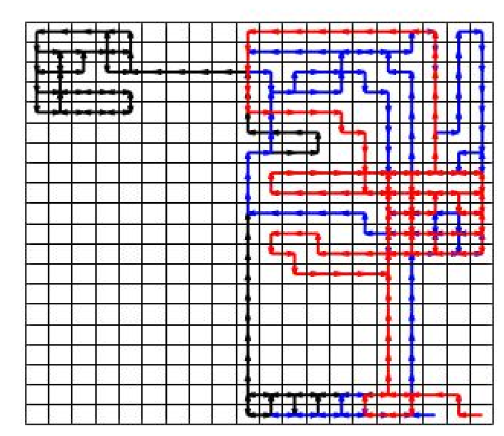
(c) Map showing the reward rate of each cell where the darker means a higher rate of reward. The area marked by blue square appears after 100 steps. The vertical strip located right of the area with blue square has the lowest rate of reward.



(d) The reward gathered using three different methods. The first one is the traditional greedy where agents move to the most rewarding cell. The second method is using Algorithm1 without incorporating nodal importance and the third method is using nodal importance.



(e) The path that agent take in their first 150 steps when they follow Algorithm1 without nodal importance



(f) The path that agent take in their first 150 steps when they follow Algorithm1 with nodal importance

Conclusion

- We proposed a solution for persistent monitoring of a finite number of inter-connected geographical nodes with the purpose of maximizing the expected value of event detection.
- By showing that the reward function is a monotone increasing and submodular set function, we laid the ground to propose a suboptimal solution with a known optimality gap.

Note: see `https://arxiv.org/pdf/1908.04425.pdf` for the extended version of our work and further discussions.

This work is supported by NSF award IIS-SAS-1724331

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