## Kia Cooperative Systems Summer High School Outreach Module 5

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## Why do we care about robot motion planning?

Regardless of the form of the robots or the task it must perform, robots must maneuver through the world.

Motion planning is the problem of finding a robot motion from a start state to a goal state in a cluttered environment to achieve various goals while avoiding collisions.

In its simplest form, the motion planning problem is:
how to move a robot from a "start" location to a "goal" location avoiding obstacles.


"His PATH-PLANNING MAY bE SUB-OPTIMAL, BUT IT'S GOT FLAIR."

## Motion Planning: Workspace

a robot described by a moving point (that is, the robot has zero size).

> A workspace $W \subset R^{2}$ or $R^{3}$, often just a rectangle;
$>$ Some obstacles $O_{1}, O_{2}, \cdots, O_{n}$;
$>\mathrm{A}$ start point $p_{\text {start }}$ and a goal point $p_{\text {goal }}$;
free workspace: $W_{\text {free }}=W \backslash\left(O_{1} \cup O_{2} \cup \cdots \cup O_{n}\right)$ : the set of points in $W$ that are outside all obstacles.

## Motion Planning Using Visibility Graph

Visibility roadmaps: the visibility graph $G=(V, E, w)$, is defined as
i. the nodes $V$ of the visibility graph are all the vertices of the polygons $O_{1}, \cdots, O_{n}$,
ii. the edges $E$ of the visibility graph are all pairs of vertices that are visibly connected. That is, given $u, v \in V$, we add the edge $\{u, v\}$ to the edge set $E$ if the straight-line segment between $u$ and $v$ is not in collision with any obstacle, and
iii. the weight of an edge $\{u, v\}$ is given by the length of the segment connecting $u$ and $v$.

environments with polygonal obstacles.


## Kinematics: motion of a robot moving on a straight line

Kinematics: the branch of mechanics concerned with the motion of objects without reference to the forces that cause the motion. In Kinematics, we characterize the relationship between displacement, velocity and acceleration of the objects.

To keep track of displacement of a robot on a 2D plane in a Cartesian coordinate system we need a reference point and 2 reference directions.


## Kinematics: motion of a robot moving on a straight line

Consider a robot moving on a straight line with constant velocity:


Displacement: $\Delta \mathrm{p}=\mathrm{x}\left(\mathrm{t}_{2}\right)-\mathrm{x}\left(\mathrm{t}_{1}\right)=\mathrm{v}\left(\mathrm{t}_{2}-\mathrm{t}_{1}\right)$

- Starting from an initial position $x(0)$ at time $t=0$, the position of our robot at the consecutive times is given by

$$
x(t)=v t+x(0)
$$

## Kinematics: motion of a robot moving on a straight line

Consider a robot moving on a straight line with constant velocity:



- Position at each time $x(t)=v t+x(0)$
- We are only interested in knowing the position at particular time $t_{i+1}=t_{i}+\Delta t, i=0,1,2, \ldots$ where $t_{0}=0$, and $\Delta t$ is the sampling time ( think of a radar which only samples the motion at particular times ). Then, the position of our robot at each is given by $x(i+1)=x\left(t_{i+1}\right)=v \Delta t+x_{i}$



## Kinematics: motion of a robot moving on a straight line

Consider a robot moving on a straight line with constant velocity:



- Position at each time $x(t)=v t+x(0)$

Robot moving on a 2 dimensional space on a straight line with constant velocity:


$$
\begin{aligned}
& \text { Starts at }\left(x_{0}, y_{0}\right) \\
& \text { Ends at }\left(x_{f}, y_{f}\right)
\end{aligned}
$$

Constant velocity along the straight line is v

- Velocity in $x$ direction $V_{x}=v \cos (\theta)$
- Velocity in $x$ direction $V_{y}=v \sin (\theta)$

Consider a robot moving on a 2 dimensional space on a straight line with constant velocity:


| Starts at $\left(x_{0}, y_{0}\right)$ |
| :--- |
| Ends at $\left(x_{f}, y_{f}\right)$ |



Constant velocity along the straight line is $v$

- Velocity in $x$ direction $V_{x}=v \cos (\theta)$
- Velocity in $x$ direction $V_{y}^{x}=v \sin (\theta)$

$$
\begin{aligned}
& \cos (\theta)=\frac{x_{f}-x_{0}}{d} \\
& \sin (\theta)=\frac{y_{f}-y_{0}}{d} \\
& d=\sqrt{\left(x_{f}-x_{0}\right)^{2}+\left(y_{f}-y_{0}\right)^{2}}
\end{aligned}
$$

- Position in $x$ direction: $x(i+1)=v \cos (\theta) \Delta t+x(i)$
- Position in y direction: $y(i+1)=v \sin (\theta) \Delta t+y(i)$



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